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A Conceptual Proof of Cramer's Rule

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THEOREM. (CRAMER'S RULE) *Let A be an invertible $n \times n$ matrix. Then the solutions x_i to the system $A\mathbf{x} = \mathbf{b}$ are given by*

$$x_i = \frac{\det(A_i)}{\det(A)}, \tag{1}$$

where A_i is the matrix obtained from A by replacing the ith column of A by $\mathbf{b}$.

Proof. The classical way to solve a linear equation system is by performing row operations: (i) add one row to another row, (ii) multiply a row with a nonzero scalar and (iii) exchange two rows. We show that the quotient in equation (1) will not change under row operations.

Under the first row operation, the values of the two determinants $\det(A_i)$ and $\det(A)$ will not change, since determinants are invariant under this row operation. Under the second row operation both determinants will gain the same factor, which cancels in the quotient. Finally, under the third row operation both determinants will switch sign, which again cancels in the quotient.

Since every invertible matrix A can be row reduced to the identity matrix, it is now enough to prove Cramer's rule for the identity matrix. However, this is a straightforward task. ∎

A Parent of Binet's Formula?

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The famous Binet formula for the Fibonacci sequence $F_1 = 1 = F_2$, $F_{n+2} = F_n + F_{n+1}$ is the identity

$$F_n = \frac{\phi^n - (-1/\phi)^n}{\sqrt{5}},$$

where ϕ is the golden ratio $(1+\sqrt{5})/2$.