Exam 1

Name: ______________________________ Section: ______________

Do not remove this answer page — you will return the whole exam. You will be allowed two hours to complete this test. You are allowed to use notes on a single piece of 8.5"X11" paper, front and back, including formulas and theorems. **You are required to turn this page in with your exam.** You may use a graphing calculator during the exam, but NO calculator with a Computer Algebra System (CAS). Absolutely no communication device use during the exam is allowed.

The exam consists of 10 multiple choice questions and 5 free response questions. Record your answers to the multiple choice questions on this page by filling in the circle corresponding to the correct answer.

Show all work to receive full credit on the free response problems. It will also help you check your answers to show work on multiple choice problems.

Multiple Choice Questions

1	A	B	C	D	E	**6**	A	B	C	D	E
2	A	B	C	D	E	**7**	A	B	C	D	E
3	A	B	C	D	E	**8**	A	B	C	D	E
4	A	B	C	D	E	**9**	A	B	C	D	E
5	A	B	C	D	E	**10**	A	B	C	D	E

Multiple Choice	11	12	13	14	15	Total Score
50	10	10	10	10	10	100

Trig Identities

- $\sin^2(x) + \cos^2(x) = 1$ and $\tan^2(x) + 1 = \sec^2(x)$
- $\sin^2(x) = \frac{1}{2}(1 - \cos(2x))$ and $\cos^2(x) = \frac{1}{2}(1 + \cos(2x))$

Multiple Choice Questions

1. (5 points) Find $\int x^2 e^x \, dx$.

   A. $e^{x^2} 2x + C$.

   B. $xe^{x^2} + 2x + C$.

   C. $x^2 e^x - 2xe^x + 2e^x + C$**.**

   D. $\frac{1}{3}x^3 e^x + C$.

   E. $x^2 e^x + 2e^x + C$.

2. (5 points) If $f(0) = 2$, $f(1) = 2$, $f'(0) = 1$ and $f'(1) = 3$, and $f''(x)$ is continuous, what is $\int_0^1 (x+1) f''(x) \, dx$?

   A. 5

   B. -1

   C. 6

   D. -4

   E. 1

3. (5 points) Find $\int \sin^2(3x)\ dx$.

   A. $\frac{1}{3}\sin^3(3x)\cos(3x) + C$.

   B. $6\sin(3x) + C$.

   C. $-\frac{1}{3}\cos^2(3x) + C$.

   D. $\frac{1}{2}x - \frac{1}{12}\sin(6x) + C$**.**

   E. $-\frac{1}{3}\sin^3(3x)\cos(3x) + C$.

4. (5 points) Which of the following is equal to the integral

$$\int \left(\frac{1}{\sqrt{16-x^2}}\right)^3\ dx$$

after making the substitution $x = 4\sin(\theta)$?

   A. $-\dfrac{1}{16}\displaystyle\int \csc^2(\theta)\ d\theta$.

   B. $\dfrac{1}{16}\displaystyle\int \sec^2(\theta)\ d\theta$**.**

   C. $\dfrac{1}{4}\displaystyle\int \sec^2(\theta)\ d\theta$.

   D. $-\dfrac{1}{4}\displaystyle\int \csc^2(\theta)\ d\theta$.

   E. $\dfrac{1}{16}\displaystyle\int \cos^2(\theta)\ d\theta$.

5. (5 points) Find the limit:

$$\lim_{n\to\infty} \frac{2n^2+5}{n^2+7}.$$

   A. 1

   B. 2

   C. -2

   D. -4

   E. 8

6. (5 points) Find $\int_1^\infty \frac{1}{x^{\frac{8}{3}}}\,dx$

A. $\frac{8}{3}$
B. $\frac{5}{3}$
C. $\frac{2}{3}$
D. $\frac{3}{8}$
E. $\frac{3}{5}$

7. (5 points) What is the form of the partial fraction decomposition of

$$\frac{x^2-2}{(x+1)^2(x^2+2)(x-1)}?$$

A. $\frac{A}{x+1}+\frac{Bx+C}{x^2+2}+\frac{Dx+E}{x^2-1}$
B. $\frac{A}{x+1}+\frac{B}{(x+1)^2}+\frac{Dx+E}{x^2+2}$
C. $\frac{A}{x+1}+\frac{B}{(x+1)^2}+\frac{C}{x-1}+\frac{Dx+E}{x^2+2}$
D. $\frac{A}{x+1}+\frac{Bx+C}{x^2-1}$
E. $\frac{A}{x+1}+\frac{B}{(x+1)^2}+\frac{C}{x-1}$

8. (5 points) Find the coefficient B in the partial fraction decomposition

$$\frac{x}{(x-1)(x^2+1)}=\frac{A}{x-1}+\frac{Bx+C}{x^2+1}$$

A. $B=\frac{1}{2}$
B. $B=-1$
C. $B=1$
D. $B=-\frac{1}{2}$
E. $B=-2$

9. (5 points) Let $f(x)$ be a function that satisfies $|f''(x)| \leq 3$ on the interval $[3, 5]$. Choose the smallest n so that we can be sure that $E_M = |M_n - \int_3^5 f(x)dx| \leq .0001$, where M_n is the midpoint approximation with n intervals.

   A. $n = 100$

   B. $n = 500$

   C. $n = 10,000$

   D. $n = 200$

   E. $n = 50$

10. (5 points) Find the Simpson's rule estimate of $\displaystyle\int_1^5 x^3\, dx$ for $n = 4$.

   A. $S_4 = \dfrac{424}{3}$

   B. $S_4 = \dfrac{324}{3}$

   C. $S_4 = 162$

   D. $S_4 = 78$

   E. $S_4 = 156$

Free Response Questions

11. (a) (2 points) Compute $\displaystyle\int x\cos(x)\ dx$

Solution: Integration by parts: Let $u = x$ so $du = dx$, and $dv = \cos(x)dx$ so $v = \sin(x)$. Then, $\int x\cos(x)dx = x\sin(x) - \int \sin(x)dx = x\sin(x) + \cos(x) + C$.

(b) (8 points) Compute $\displaystyle\int_0^{\frac{\pi}{2}} x\cos(x)\ dx$

Solution: From part (a), an anti-derivative is $F(x) = x\sin(x) + \cos(x)$, so the Fundamental Theorem of Calculus gives: $\int_0^{\pi/2} x\cos(x)dx = F(\frac{\pi}{2}) - F(0) = \left[\frac{\pi}{2}\sin\left(\frac{\pi}{2}\right) + \cos\left(\frac{\pi}{2}\right)\right] - \cos(0) = \frac{\pi}{2} - 1$.

12. (10 points) Compute $\int \sqrt{4-x^2}\,dx$. You must simplify your answer.

Solution: Let $J(x)$ be the integral. Use the trig substitution $x = 2\sin\theta$ so $\sqrt{4-x^2} = 2\sqrt{1-\sin^2\theta} = 2\cos\theta$. We take $\theta \in [-\pi/2, \pi/2]$ so the cosine is positive. Since $dx = 2\cos\theta\,d\theta$, we get $J(x) = 4\int \cos^2\theta\,d\theta = 2\int[1+\cos(2\theta)]d\theta = 2[\theta+\frac{1}{2}\sin(2\theta)]+C$. Solving the substitution, $\theta = \sin^{-1}(x/2)$. Since $\sin(2\theta) = 2\sin\theta\cos\theta$, and $\cos^2\theta = 1-(x/2)^2$, we get $J(x) = 2[\sin^{-1}(x/2) + (x/4)\sqrt{4-x^2}] + C$.

13. (10 points) Compute $\displaystyle\int_1^\infty x^2 e^{-x^3}\ dx$.

Solution: The improper integral is defined by $\lim_{M\to\infty}\int_1^M x^2e^{-x^3}\ dx$. Letting $u = x^3$, the indefinite integral is $\int x^2 e^{-x^3}\ dx = (1/3)\int e^{-u}du = -(1/3)e^{-x^3} + C$. The definite integral is $\int_1^M x^2e^{-x^3}\ dx = \frac{1}{3}[\frac{1}{e} - e^{-M^3}]$. Since $e^{-M^3} \to 0$ as $M \to \infty$, we get $\lim_{M\to\infty}\int_0^M x^2e^{-x^3}\ dx = \frac{1}{3e}$.

14. (10 points) Using the method of partial fractions, compute

$$\int \frac{x+1}{(x+2)(x^2+1)}\,dx.$$

Solution:

The partial fraction is:

$$\frac{x+1}{(x+2)(x^2+1)} = \frac{1}{x+2} + \frac{Bx+C}{x^2+1},$$

Solving $x+1 = A(x^2+1) + (x+2)(Bx+C) = (A+B)x^2 + (2B+C)x + (A+2C)$ leading to $A = -\frac{1}{5}$, $B = \frac{1}{5}$, and $C = \frac{3}{5}$. The integral becomes

$$-\frac{1}{5}\int \frac{dx}{x+2} + \frac{1}{5}\int \frac{x}{x^2+1}dx + \frac{3}{5}\int \frac{dx}{x^2+1},$$

leading to

$$-\frac{1}{5}\log|x+2| + \frac{1}{10}\log(x^2+1) + \frac{3}{5}\tan^{-1}(x) + C.$$

15. (a) (5 points) Use the midpoint rule to estimate the integral

$$\int_1^9 \frac{1}{x^2}\, dx$$

Use four intervals (ie find M_4).

Solution: $M_4 = 0.7118$. Midpoint rule for $f(x) = \frac{1}{x^2}$ and $n = 4$, with $b - a = 9 - 1 = 8$, so $\Delta = 2$. The endpoints are $x_0 = a = 1, x_2 = 3, x_3 = 7, x_4 = b = 9$. The midpoints are $x_1^* = 2, x_2^* = 4, x_3^* = 6, x_4^* = 8$. Then, the approximation M_4 is

$$M_4 = 2[f(2) + f(4) + f(6) + f(8)].$$

(b) (5 points) Use the trapezoid rule to estimate the integral

$$\int_1^9 \frac{1}{x^2} dx$$

Use four intervals (ie find T_4).

Solution: $T_4 = 0.6777$. Trapezoidal rule for $f(x) = \frac{1}{x^2}$ and $n = 4$, with $b - a = 9 - 1 = 8$, so $\Delta = 2$. The endpoints are $x_0 = a = 1, x_2 = 3, x_3 = 7, x_4 = b = 9$. Then, the approximation T_4 is

$$T_4 = 2[\frac{1}{2}f(1) + f(3) + f(5) + f(7) + \frac{1}{2}f(9)].$$