Exam 1 a

Record the correct answer to the following problem on the front page of this exam.

(1) Evaluate the integral $\int x \sin(x) dx$.

A) $\frac{-x^2}{2}\cos(x)$ +C

B) $x\sin(x) - \cos(x)$ +C

C) $\frac{-\sin^2(x)}{2}$ +C

D) $-x\cos(x) + \sin(x) + C$

E) $x\cos(x) + \sin(x)$+C

$u = x \quad v' = \sin(x)$

$u' = 1 \quad v = -\cos(x)$

$= -x\cos(x) + \int \cos(x)\, dx$

$= -x\cos(x) + \sin(x) + C$

(2) If a force F $= F(x)$ in Newtons is given by the graph shown, find the work done by the force in Joules in moving a particle from $x = 0$ to $x = 4$, where x is in meters.

A) 2

B) 3

C) 4

D) 5

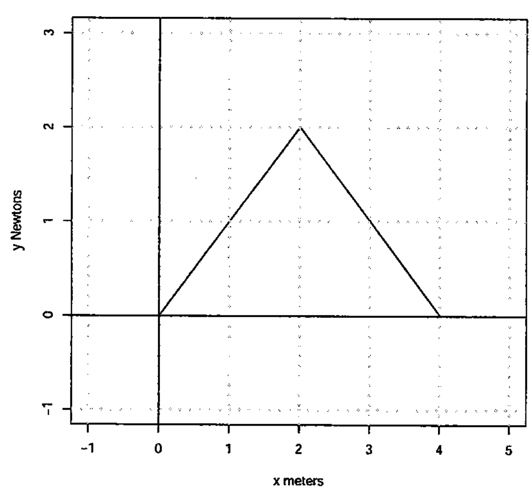
E) 6

$W = \int F(x)\, dx$

$=$ area under $F(x)$

$= \frac{1}{2} b h =$

$\frac{1}{2}(4)(2) = 4$



Record the correct answer to the following problem on the front page of this exam.

(3) Given a solid whose base is a circle in the x, y plane centered at the origin with radius 2, and whose cross sections taken perpendicular to the x-axis are squares. Find the area of a cross section taken at $x = x_0$.

$x^2+y^2=4$

A) $4(4 - x_0^2)$

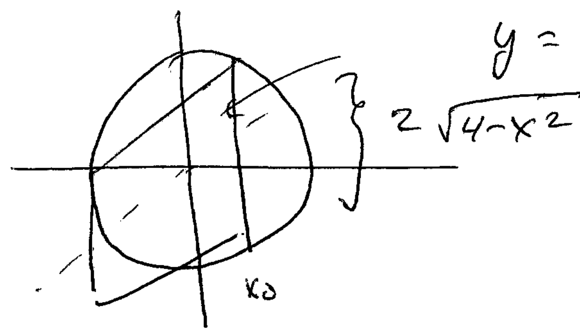
B) $2\pi(4 - x_0^2)$

C) 16π

D) $2(x_0^2 - 4)$

E) $4\sqrt{4 - x_0^2}$

$y = \sqrt{4-x^2}$



at x_0

height of square $= 2\sqrt{4-x_0^2}$

area $= \left(2\sqrt{4-x_0^2}\right)^2$

$= 4\left(4-x_0^2\right)$

(4) To evaluate $\int x^3 e^{x^2} dx$ by parts, what is the correct substitution?

A) $u(x) = x^2, v'(x) = xe^{x^2}$

B) $u(x) = x^3, v'(x) = e^{x^2}$

C) $u(x) = x, v'(x) = e^{x^2}$

D) $u(x) = e^{x^2}, v'(x) = x^2$

E) $u(x) = 1, v'(x) = x^3 e^x$

$u = x^2$ $\quad v' = xe^{-x^2}$

$u' = 2x\,dx$ $\quad v = -\frac{1}{2} e^{-x^2}$

$= -\frac{x^2}{2} e^{-x^2} + \int x e^{-x^2} dx$

$= -\frac{x^2}{2} e^{-x} + -\frac{1}{2} e^{-x^2} + C$

$\cancel{u = x}$ $\quad \cancel{v' = e^{x^2}}$

$\cancel{u' = 1}$ $\quad \cancel{v =}$

Record the correct answer to the following problem on the front page of this exam.

(5) To evaluate the integral $\int \frac{x^2}{(x^2-4)^{3/2}}dx$ one would use which substitution?

A) $x = 2\tan(\theta)$

B) $x = 2\sin(\theta)$

C) $x = 2\sec(\theta)$

D) $u = x^2 - 1$

E) $x = 4\sin(\theta)$

$X = 2\sec(\theta)$

$dx = 2\sec(\theta)\tan(\theta)$

$(x^2-4)^{3/2} = (4\sec^2\theta - 4)^{3/2}$

$= (4\tan^2\theta)^{3/2}$

$= 8\tan^3\theta$

$= \frac{(4\sec^2\theta)(2\sec(\theta)\tan(\theta))\,d\theta}{8\tan^3\theta}$

$= \int \frac{\sec^3 d\theta}{\tan^2(\theta)}$ ⇐ solvable so works

(6) Evaluate the integral $\int \sin(2x)\cos(2x)dx$.

A) $\frac{1}{2}\cos^2(2x)$+C

B) $\tan(2x)$+C

C) $\sec(2x)\tan(2x)$+C

D) $\frac{1}{4}\sin^2(2x)$+C

E) $\cos(2x)\sin(2x)$+C

$u = \sin(2x) \qquad du = 2\cos(2x)$

$= \int u \frac{du}{2}$

$= \frac{1}{2}\frac{\sin^2(2x)}{2} + c$

Free Response Questions: Show your work!

(7) Evaluate the integral
$$\int \sec^3(x)\tan^3(x)dx.$$

$$= \int \sec^2(x)\tan^2(x)\sec(x)\tan(x)\,dx$$

$$= \int \sec^2(x)\left(\sec^2(x)-1\right)\sec(x)\tan(x)\,dx$$

$$= \int \left(\underbrace{\sec^4(x)}_{u^4} - \underbrace{\sec^2(x)}_{u^2}\right)\underbrace{\sec(x)\tan(x)\,dx}_{du}$$

$$= \frac{\sec^5(x)}{5} - \frac{\sec^3(x)}{2} + C$$

Free Response Questions: Show your work!

(8) Evaluate the integral

$$\int \frac{x^2}{\sqrt{9-x^2}}dx.$$

$x = 3\sin(\theta) \qquad dx = 3\cos(\theta)\, d\theta$

$\sqrt{9-x^2} = \sqrt{9 - 9\sin^2(\theta)} = 3\cos(\theta)$

$$= \int \frac{9\sin^2(\theta)\, \cancel{3\cos(\theta)}}{\cancel{3\cos(\theta)}} d\theta$$

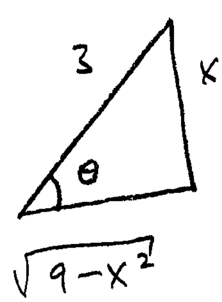
$$= 9 \int \sin^2(\theta)\, d\theta$$

$$= \frac{9}{2} \int 1 - \cos(2\theta)\, d\theta$$

$$= \frac{9}{2}\left(\theta - \frac{\sin(2\theta)}{2}\right) + C$$

$\frac{x}{3} = \sin(\theta) \Rightarrow \theta = \arcsin(x/3)$

$\cos(\theta) = \frac{\sqrt{9-x^2}}{3}$



$$\frac{9}{2}\left(\theta - \sin(\theta)\cos(\theta)\right) + C$$

$$\frac{9}{2}\left(\arcsin(x/3) - \frac{x}{3}\left(\frac{\sqrt{9-x^2}}{3}\right)\right)$$

(9) Find the average of the function $f(x) = x^2\sin(x)$ over the interval $0 \le x \le \pi$.

$\int x^2 \sin(x)\,dx =$ $u = x^2$ $v' = \sin(x)$

$u' = 2x \leftarrow v = -\cos(x)$

$= -x^2\cos(x) + \int 2x\cos(x)\,dx$

$u = 2x$ $v' = \cos(x)$

$u' = 2$ $v = \sin(x)$

$= -x^2\cos(x) + 2x\sin(x) - 2\int \sin(x)\,dx$

$= -x^2\cos(x) + 2x\sin(x) + 2\cos(x)$

$\frac{1}{\pi - 0}\int_0^{\pi} x^2\sin(x)\,dx =$

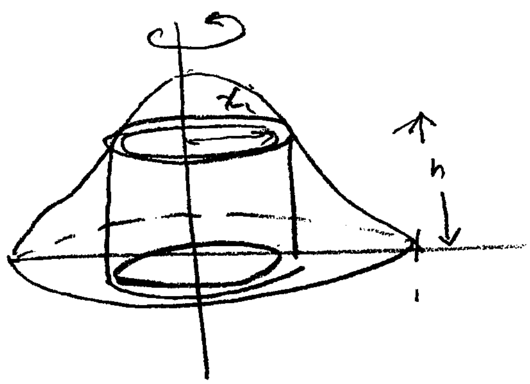
$\frac{1}{\pi}\left(-x^2\cos(x) + 2x\sin(x) + 2\cos(x)\Big|_0^{\pi}\right)$

$= \frac{1}{\pi}\left(\pi^2 + 0 - 2 - (2)\right)$

$= \frac{\pi^2 - 4}{\pi} = 1.86835$

Free Response Questions: Show your work!

(10) Find the volume of the solid obtained by revolving the area under the curve $y = (x^2 - 1)^{2/3}$, and above the x-axis for $0 \le x \le 1$, about the y-axis. (Hint: Use the method of shells)



$$V_i = 2\pi x_i (x_i^2 - 1)^{2/3} \Delta x$$

$$V = 2\pi \underbrace{\int_0^1 x \underbrace{(x^2-1)}_{u}^{2/3} dx}_{\frac{du}{2}}$$

$u = x^2 - 1$

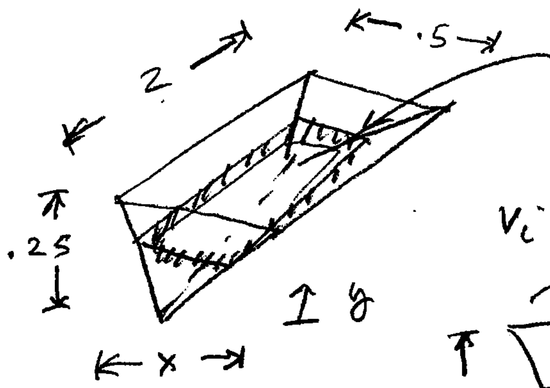
$du = 2x\, dx$

$$= \frac{2\pi}{2} (x^2-1)^{5/3} \left(\tfrac{3}{5}\right)$$

$$= \pi \frac{3}{5} (x^2-1)^{5/3} \Big|_0^1$$

$$= 0 - \left(-\frac{3\pi}{5}(-1)^{5/3}\right) = \frac{3\pi}{5}$$

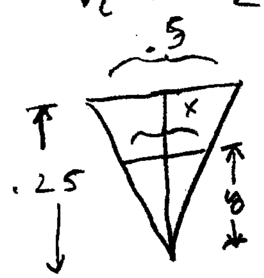
(11) Find the work done in emptying a trough by pumping the water from the top. The trough is 2 meters long, 0.5 meters wide at the top, and 0.25 meters deep and has a cross section that is an isosceles triangle. The density of water is 1000 kg/m^3.



i'th slice rectangle

Volume

$V_i = 2\,(x_i)\,\Delta y$



$\frac{x/2}{y} = \frac{.5/2}{.25}$

$x_i = 2y_i$

$V_i = 2\,(2y_i)\,\Delta y$

$\text{Mass}_i = 1000\,(4\,y_i)\,\Delta y$

$\text{Force}_i = 4000\,(9.8)\,y_i\,\Delta y$

$\text{Work}_i = 4000\,(9.8)\,y_i\,\underbrace{(.25 - y_i)}_{\text{distance to top}}\,\Delta y$

$\text{total work} = 9.8(4000)\int_0^{.25} y\,(.25 - y)\,dy$

$= 9.8\,(4000)\left(\frac{.25\,y^2}{2} - \frac{y^3}{3}\right)\Big|_0^{.25}$

$= 9.8(4000)\left(\frac{.25^3}{2} - \frac{.25^3}{3}\right) = 102.08333\ \text{J}$