MA 114 — Calculus II Spring 2015
Exam 1 Feb. 10, 2015

Name: ______________________________

Section: ____________________________

Last 4 digits of student ID #: ________

- No books or notes may be used.
- Turn off all your electronic devices and do not wear ear-plugs during the exam.
- You may use a calculator, but not one which has symbolic manipulation capabilities or a QWERTY keyboard.
- Additional blank sheets for scratch work are available upon request.
- **Multiple Choice Questions:**
  Record your answers on the right of this cover page by marking the box corresponding to the correct answer.
- **Free Response Questions:**
  Show all your work on the page of the problem. Clearly indicate your answer and the reasoning used to arrive at that answer.

Multiple Choice Answers

Question					
1	A	B	C	D	E
2	A	B	C	D	E
3	A	B	C	D	E
4	A	B	C	D	E
5	A	B	C	D	E
6	A	B	C	D	E
7	A	B	C	D	E

Exam Scores

Question	Score	Total
MC		28
8		13
9		15
10		14
11		15
12		15
Total		100

Unsupported answers for the free response questions may not receive credit!

Record the correct answer to the following problems on the front page of this exam.

1. Let a, b be real numbers and consider the integral $\int (ax^2 + b)\cos(x)dx$. Using integration by parts leads to which of the following expressions?

$u = ax^2 + b$
$u' = 2ax$
$v' = \cos(x)$
$v = \sin x$

$uv - \int u'v\,dx$

$= (ax^2+b)\sin x - 2a\int x\sin x\,dx$

A. $(ax^2 + b)\cos(x) - 2a \int x\cos(x)dx.$

B. $(ax^2 + b)\cos(x) + 2a \int x\sin(x)dx.$

C. $(ax^2 + b)\sin(x) - 2a \int x\sin(x)dx.$

D. $2ax\sin(x) - 2\int (ax^2 + b)\sin(x)dx.$

E. $2ax\cos(x) + 2\int (ax^2 + b)\sin(x)dx.$

2. Evaluate the series $\sum_{n=0}^{\infty} 2^{4-3n}$.

$= \sum_{n=0}^{\infty} 2^4 \cdot \left(\frac{1}{8}\right)^n = 16 \cdot \sum_{n=0}^{\infty} \left(\frac{1}{8}\right)^n$

$= 16 \cdot \frac{1}{1-\frac{1}{8}}$

$= \frac{128}{7}$

A. $\frac{128}{7}$.

B. 16.

C. $\frac{8}{7}$.

D. 0.

E. The series diverges.

Record the correct answer to the following problems on the front page of this exam.

3. Let $a > 3$ be a fixed number. Evaluate the improper integral $\int_a^\infty \frac{1}{(x-3)^2}dx$.

$$\lim_{R\to\infty} \int_a^R \frac{1}{(x-3)^2}\,dx = \lim_{R\to\infty} \left(-(x-3)^{-1} \Big|_a^R \right)$$

$$= \lim_{R\to\infty} \left(\frac{1}{a-3} - \underbrace{\frac{1}{R-3}}_{\to 0} \right) = \frac{1}{a-3}$$

A. $\frac{1}{a-3}$.

B. 0.

C. ∞.

D. $\frac{1}{(a+3)^2}$.

E. $\frac{1}{(a-3)^3}$.

4. Which of the following is true for a series $\sum_{n=1}^{\infty} a_n$? There is only one correct answer.

A. If the series is convergent, then it is also absolutely convergent.

B. If the series is alternating, then it is convergent.

C. If $\lim_{n\to\infty} a_n = L$, then $\sum_{n=1}^{\infty} a_n = L$.

D. If $\lim_{n\to\infty} a_n = 0$, then the series converges.

E. If $\lim_{n\to\infty} a_n \neq 0$, then the series diverges.

Record the correct answer to the following problems on the front page of this exam.

5. Which of the following is true for the sequence $\left\{\frac{1}{\sqrt[5]{n^2+1}}\right\}$? There is only one correct answer.

A. The sequence is increasing and divergent.

B. The sequence is increasing and convergent.

C. The sequence is decreasing and divergent.

D. The sequence is decreasing and convergent.

E. The sequence is alternating and divergent.

6. Consider the series $\sum_{n=1}^{\infty} \frac{1}{2^n + 5n - 2}$. Using the comparison test with the series $\sum_{n=1}^{\infty} \frac{1}{2^n}$ leads to the following result. There is only one correct answer.

A. The series converges absolutely.

$2^n + \underbrace{5n-2}_{>0} > 2^n$

B. The series converges conditionally.

so $\frac{1}{2^n+5n-2} \leq \frac{1}{2^n}$

C. The series diverges.

$\sum \frac{1}{2^n}$ converges.

D. The test is inconclusive.

E. The test is not applicable for $a_n = \frac{1}{2^n + 5n - 2}$ and $b_n = \frac{1}{2^n}$.

Record the correct answer to the following problems on the front page of this exam.

7. Find the radius of convergence of the power series $\sum_{n=0}^{\infty} \frac{(-1)^n n}{4^n}(x-2)^n$.

A. $\frac{1}{4}$.

B. 2.

C. 4.

D. 16.

E. ∞.

$$\left|\frac{a_{n+1}}{a_n}\right| = \left|\frac{(n+1)\cdot(x-2)^{n+1} \cdot 4^n}{4^{n+1} \cdot n\,(x-2)^n}\right|$$

$$= \left|\frac{1}{4}\underbrace{\frac{n+1}{n}}_{\to 1}(x-2)\right| \xrightarrow[n\to\infty]{} \frac{1}{4}|x-2|$$

Series conv. abs. if $\frac{1}{4}|x-2| < 1$, so
$|x-2| < 4$

Series diverges if $\frac{1}{4}|x-2| > 1$, so $|x-2| > 4$.

Free Response Questions: Show your work!

8. Evaluate the improper integral
$$\int_0^\infty xe^{-2x}\,dx.$$

You have to use proper notation and re-write the integral as a limit.

$$\int_0^\infty x e^{-2x}\,dx = \lim_{R\to\infty} \int_0^R x e^{-2x}\,dx$$

$$\int_0^R \underbrace{x}_{u} \underbrace{e^{-2x}}_{v'}\,dx \underset{\substack{u'=1 \\ v=-\frac{1}{2}e^{-2x}}}{=} -\frac{1}{2} x e^{-2x}\Big|_0^R + \frac{1}{2}\int_0^R e^{-2x}\,dx$$

$$= \left[-\frac{1}{2}xe^{-2x} - \frac{1}{4}e^{-2x}\right]_0^R$$

$$= -\frac{R}{2e^{2R}} - \underbrace{\frac{1}{4e^{2R}}}_{\xrightarrow{R\to\infty} 0} + \frac{1}{4}$$

$$\lim_{x\to\infty} \frac{x}{2e^{2x}} \underset{\boxed{\frac{\infty}{\infty}}}{\overset{\text{l'Hospital}}{=}} \lim_{x\to\infty} \frac{1}{4e^{2x}} = 0.$$

$$\text{So, } \int_0^\infty x e^{-2x}\,dx = \lim_{R\to\infty}\left(\frac{1}{4} - \frac{R}{2e^{2R}} - \frac{1}{4e^{2R}}\right) = \underline{\underline{\frac{1}{4}}}$$

Free Response Questions: Show your work!

9. Evaluate the following integrals.

(a) $\int \underbrace{\frac{16}{x^3}}_{v'} \underbrace{\ln(x)}_{u} dx.$

$u' = \frac{1}{x}$

$v = -8x^{-2}$

$- \frac{8}{x^2} \ln x + 8 \int x^{-3} dx$

$= -\frac{8}{x^2} \ln x - 4x^{-2} + C$

$= -\frac{8}{x^2} \ln x - \frac{4}{x^2} + C$

(b) $\int_2^5 \frac{1}{x-2} dx. = \lim_{R \to 2^+} \int_R^5 \frac{1}{x-2} dx$

$= \lim_{R \to 2^+} \left(\ln(x-2) \Big|_R^5 \right)$

$= \lim_{R \to 2^+} \left(\ln(3) - \underbrace{\ln(\underbrace{R-2}_{\to 0})}_{\to -\infty} \right) = +\infty$

10. Use the integral test to determine whether the series

$$\sum_{n=1}^{\infty} \frac{6n}{(n^2+2)^4}$$

converges or diverges. You have to verify all assumptions of the test.

Set $f(x) = \frac{6x}{(x^2+2)^4}$. Then

$f(x) > 0$ and continuous on $[1,\infty)$.

$f(x)$ is decreasing on $[1,\infty)$ b/c

$$f'(x) = \frac{6(x^2+2)^{\cancel{4}} - 6x \cdot 4\cancel{(x^2+2)^3} \cdot 2x}{(x^2+2)^{\cancel{8}\,5}}$$

$$= \frac{6x^2+12-48x^2}{(x^2+2)^5} = \frac{-42x^2+12}{(x^2+2)^5}$$

< 0 for $x \geq 1$.

Thus we may apply the integral test:

$$\int_1^\infty \frac{6x}{(x^2+2)^4}\,dx \underset{du = 2x\,dx}{\overset{u = x^2+2}{=\!=\!=}} 3\int_3^\infty u^{-4}\,du$$

$$= \lim_{R\to\infty}\left[\left(-u^{-3}\right)\Big|_3^R\right] = \lim_{R\to\infty}\left[\frac{1}{27} - \underbrace{\frac{1}{R^3}}_{\to 0}\right]$$

$$= \frac{1}{27}.$$

So, the integral converges and therefore the series converges.

Free Response Questions: Show your work!

11. Determine whether the following series converges or diverges. Make sure to state all tests that you use and to show all work required to apply the test.

(a) $\underbrace{\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^2+3n+6}}}_{a_n}$ Use Limit Comparison Test:

with $b_n = \frac{1}{n}$. Then $\frac{a_n}{b_n} = \frac{n}{\sqrt{n^2+3n+6}}$

$= \sqrt{\underbrace{\frac{n^2}{n^2+3n+6}}_{\longrightarrow 1}} \xrightarrow[n\to\infty]{} 1.$

Since $\sum_{n=1}^{\infty} b_n = \sum_{n=1}^{\infty} \frac{1}{n}$ diverges, so does $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^2+3n+6}}$ by the LCT.

(b) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\ln(n^2+6)} = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{\ln(n^2+6)}$.

Use Leibniz test:

$a_n = \frac{1}{\ln(n^2+6)} > 0$, decreasing b/c (n^2+6) is increasing and $\ln(x)$ is increasing. $\lim_{n\to\infty} a_n = 0$ b/c

$\lim_{n\to\infty} \ln(n^2+6) = \infty.$

Thus by Leibniz test: the series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\ln(n^2+6)}$ converges.

12. Consider the power series $\sum_{n=1}^{\infty} \frac{x^n}{n \cdot 6^n}$.

(a) Find the radius of convergence.

$$\left|\frac{a_{n+1}}{a_n}\right| = \left|\frac{x^{n+1} \cdot n \cdot 6^n}{(n+1)\cdot 6^{n+1} \cdot x^n}\right| = \left|\underbrace{\frac{n}{n+1}}_{\to 1} \cdot \frac{1}{6} \cdot x\right|$$

$$\xrightarrow[n \to \infty]{} \frac{1}{6}|x|.$$

The series converges abs. if $\frac{1}{6}|x| < 1$ and diverges if $\frac{1}{6}|x| > 1$.

$$\frac{1}{6}|x| < 1 \iff |x| < 6.$$

So, radius of convergence is $\boxed{R = 6}$

(b) Find the interval of convergence.

Test the endpoints of the interval $[-6, 6]$:

$\underline{x = -6}$: $\sum_{n=1}^{\infty} \frac{(-6)^n}{n \cdot 6^n} = \sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ converges.

$\underline{x = 6}$: $\sum_{n=1}^{\infty} \frac{6^n}{n \cdot 6^n} = \sum_{n=1}^{\infty} \frac{1}{n}$ diverges.

The interval of convergence is

$\boxed{[-6, 6)}$