Math 114 Exam 1

Name: ______________________________ Section: __________

Do not remove this answer page — you will return the whole exam. You will be allowed two hours to complete this test. No books or notes may be used. You may use a graphing calculator during the exam, but NO calculator with a Computer Algebra System (CAS) or a QWERTY keyboard is permitted. Absolutely no cell phone use during the exam is allowed.

The exam consists of 10 multiple choice questions and 5 free response questions. Record your answers to the multiple choice questions on this page by filling in the circle corresponding to the correct answer.

Show all work to receive full credit on the free response problems. The wise student will show work for the multiple choice problems as well.

Multiple Choice Questions

1 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ　**6** Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ

2 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ　**7** Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ

3 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ　**8** Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ

4 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ　**9** Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ

5 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ　**10** Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ

Multiple Choice	11	12	13	14	15	Total Score
50	10	10	10	10	10	100

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Multiple Choice Questions

1. (5 points) Which trig substitution should be used to find $\int \frac{1}{4+x^2}\,dx$?

   A. $x = 4\tan\theta$

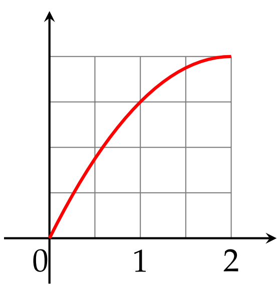
   B. $x = 2\tan\theta$

   C. $x = 2\sin\theta$

   D. $x = 4\sin\theta$

   E. $x = \sin(2\theta)$

2. (5 points) The left endpoint method (L_n), the right endpoint method (R_n), and the Trapezoid method (T_n) are used to estimate $I = \int_0^2 f(x)\,dx$ where the graph of $f(x)$ is as shown. Which of the following is correct for a given n?



   A. L_n overestimates I, T_n underestimates I, and R_n underestimates I

   B. L_n and R_n underestimate I, and T_n overestimates I

   C. L_n and T_n underestimate I, but R_n overestimates I

   D. L_n and T_n overestimate I, and R_n underestimates I

   E. L_n, R_n and T_n all overestimate I

3. (5 points) For what values of p does the improper integral

$$\int_1^\infty \frac{1}{x^{2p}}\,dx$$

   converge?

   A. $p \leq 1$

   B. $p \geq 1/2$

   C. $p \leq 1/2$

   D. $p > 1/2$

   E. $p < 1$

4. (5 points) The partial fraction decomposition of $\frac{1}{x^2+x^4}$ is

A. $\frac{A}{x}+\frac{B}{x^2}+\frac{C}{x+1}+\frac{D}{x-1}$

B. $\frac{A}{x}+\frac{B}{x^2}+\frac{C}{x+1}+\frac{D}{(x+1)^2}$

C. $\frac{A}{x}+\frac{B}{x^2}+\frac{Cx+D}{x^2+1}$

D. $\frac{A}{x}+\frac{B}{x^2}+\frac{C}{(x+1)^2}$

E. $\frac{Ax+B}{x^2}+\frac{C}{(x-1)^2}$

5. (5 points) If $x = \sin(u)$ and $-\pi/2 \le u \le \pi/2$, find $\cot(u)$.

A. $\sqrt{1-x^2}$

B. $\sqrt{1-x^2}/x$

C. $1/\sqrt{1-x^2}$

D. $x/\sqrt{1-x^2}$

E. $1/x$

6. (5 points) If we substitute $x = 4\sin u$ with $-\pi/2 \le \theta \le \pi/2$ in the integral

$$\int x\sqrt{16-x^2}\,dx,$$

we obtain

A. $\int 64\sin^2(u)\cos(u)\,du$

B. $\int 16\sin^2(u)\cos(u)\,du$

C. $\int 16\sin(u)\cos^2(u)\,du$

D. $\int 64\sin(u)\cos^2(u)\,du$

E. $\int 64\sin^2(u)\cos^2(u)\,du$

7. (5 points) How large should we take n in the Trapezoid rule in order to approximate $\int_1^2 (1/x)\,dx$ to within 0.0001? Recall that the error E_T made in applying the Trapezoid rule T_n to compute $\int_a^b f(x)\,dx$ obeys the bound

$$E_T \leq \frac{K(b-a)^3}{12n^2}$$

where K is an upper bound for $f''(x)$ on $[a,b]$.

A. $n = 41$ or larger

B. $n = 40$ or less

C. $n = 20$

D. $n = 10$

E. $n = 5$

8. (5 points) Evaluate $\displaystyle\int \frac{5x+1}{(2x+1)(x-1)}\,dx$

A. $\ln|2x+1| + \ln|x-1| + C$

B. $\frac{1}{2}\ln|2x+1| + 2\ln|x-1| + C$

C. $\frac{1}{5}\ln|2x+1| + \ln|x-1| + C$

D. $2\ln|2x+1| + \frac{1}{2}\ln|x-1| + C$

E. $\frac{1}{2}\ln|2x+1| + \frac{1}{2}\ln|x-1| + C$

9. Evaluate $\displaystyle\int x\cos x\,dx$

A. $x^2\cos x + x\sin x + C$

B. $x\cos x + \sin x + C$

C. $x\sin x + \cos x + C$

D. $x^2\sin x + C$

E. $x^2\cos x + C$

10. Consider the integral

$$\int_e^\infty \frac{1}{x(\ln x)^2}\,dx.$$

Which of the following statements is correct?

A. The integral is divergent

B. The integral is convergent and its value is 2

C. The integral is convergent and its value is 1

D. The integral is convergent and its value is $1/e$

E. None of these

Free Response Questions

11. (10 points) Compute $\int \frac{10}{(x-1)(x^2+9)}\,dx$

Solution: First, the partial fraction decomposition is

$$\frac{10}{(x-1)(x^2+9)} = \frac{-x-1}{x^2+9} + \frac{1}{x-1}$$

Hence

$$\int \frac{10}{(x-1)(x^2+9)}\,dx = \int \left[\frac{-x}{x^2+9} + \frac{-1}{x^2+9} + \frac{1}{x-1}\right] dx$$
$$= -\frac{1}{2}\ln(x^2+9) - \frac{1}{3}\arctan\left(\frac{x}{3}\right) + \ln|x-1| + C$$

Scoring:

(i) Partial Fraction Decomposition (6 points total):

3 points for correct form of PFD; 3 points for correct values of coefficients;.

(ii) Integration (4 points total) :

ECF allowed from PFD; 1 point for each correct term, 1 point for $+C$

12. (10 points) The following table shows the speedometer reading from a car in 1 minute intervals. Use Simpson's rule to estimate the distance travelled by the car over the 10 minute period. Be careful to make a consistent choice of units and be sure to show your work.

t (min)	0	1	2	3	4	5	6	7	8	9	10
v (mi/h)	40	42	45	49	52	54	56	57	57	55	56

Solution: Take $h = 1/60$ (in units of hours), so $h/3 = 1/180$. By Simpson's rule

$$D \approx \frac{1}{180}(40 + 4\cdot 42 + 2\cdot 45 + 4\cdot 49 + 2\cdot 52 + 4\cdot 54 + 2\cdot 56 + 4\cdot 57 + 2\cdot 57 + 4\cdot 55 + 56)$$
$$\approx 8.6 \text{ mi}$$

(Out to four decimal places one gets 8.5778)

Scoring:

Correct units and value of h (2 points); Correct weights (1 points); Correct use of data (2 points); Correct numerical answer (4 points)

13. (10 points) Compute the definite integral $\int_0^{\pi/2} \sin^2 x \cos^3 x\, dx$.

Solution: There are several right ways to work this problem–here's one of them. Let $u = \sin x$ so $du = \cos x\, dx$. Note that $u = 0$ when $x = 0$ and $u = 1$ when $x = \pi/2$. Then

$$\int_0^{\pi/2} \sin^2 x \cos^3 x\, dx = \int_0^1 u^2(1-u^2)\, du$$
$$= \left[\frac{u^3}{3} - \frac{u^5}{5}\right]\Big|_0^1$$
$$= \frac{2}{15}$$

Scoring:

Valid u-substitution (2 points),

correct substitution (2 points integrand, 2 points limits),

correct antiderivative (2 points),

correct answer (2 points)

14. (a) (5 points) Use integration by parts to compute the indefinite integral

$$\int x^2 e^{-x}\,dx$$

Solution:

$$\begin{aligned}\int x^2e^{-x}\,dx &= -x^2e^{-x} + \int 2xe^{-x}\,dx \\ &= -x^2e^{-x} - 2xe^{-x} + \int 2e^{-x}\,dx \\ &= -(x^2+2x+2)e^{-x} + C\end{aligned}$$

Scoring:

First integration by parts (2 points);
second integration by parts (2 points);
correct answer (1 point)

(b) (5 points) Determine whether the improper integral

$$\int_0^\infty x^2e^{-x}\,dx.$$

converges and if so find its value. Recall that $\lim_{x\to\infty} x^2e^{-x} = \lim_{x\to\infty} xe^{-x} = 0$ by L'Hospital's rule.

Solution:

$$\begin{aligned}\int_0^\infty x^2e^{-x}\,dx &= \lim_{R\to\infty}\left[-(x^2+2x+2)e^{-x}\right]\Big|_0^R \\ &= \lim_{R\to\infty}\left[2-(R^2+2R+2)e^{-R}\right] \\ &= 2.\end{aligned}$$

Scoring:

Express improper integral as a limit (1 point);
evaluate antiderivative correctly, ECF allowed from (a) (2 points);
evaluate limit correctly (2 points)

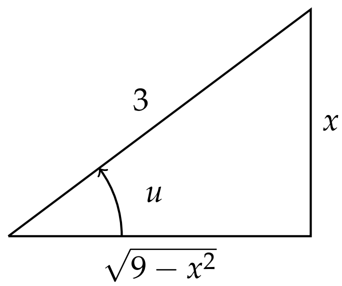
15. (10 points) Using trig substitution, evaluate the indefinite integral

$$\int \frac{x^2}{\sqrt{9-x^2}}\,dx.$$

Solution:

Let $x = 3\sin u$. Then:

$$dx = 3\cos u\,du$$
$$\sqrt{9-x^2} = 3\cos u$$



Hence

$$\begin{aligned}\int \frac{x^2}{\sqrt{9-x^2}}\,dx &= \int \frac{9\sin^2(u)\cdot 3\cos(u)}{3\cos(u)}\,du \\ &= \int 9\sin^2 u\,du \\ &= \int \frac{9}{2}\left(1-\cos(2u)\right)\,du \\ &= \frac{9}{2}u - \frac{9}{4}\sin(2u) + C \\ &= \frac{9}{2}u - \frac{9}{2}\sin(u)\cos(u) + C \\ &= \frac{9}{2}\arcsin\left(\frac{x}{3}\right) - \frac{1}{2}x\sqrt{9-x^2} + C\end{aligned}$$

Scoring:

Correct choice of substitution $x = 3\sin u$ (2 points);

Correct substitution for x^2 and $\sqrt{9-x^2}$ (2 points);

Correct substitution for dx; (1 point);

Correct computation of u-integral (2 points);

Correct conversion back to a function of x (3 points)