Problem 1.
Let a, b be real numbers and consider the integral $\int (ax^2 + b)\cos(x)\,dx$. Using integration by parts will lead to which of the following expressions?

- A. $(ax^2+b)\cos(x)+2a\int x\sin(x)\,dx$
- B. $2ax\cos(x)+2\int (ax^2+b)\sin(x)\,dx$
- C. $(ax^2+b)\sin(x)-2a\int x\sin(x)\,dx$
- D. $(ax^2+b)\cos(x)-2a\int x\cos(x)\,dx$
- E. $2ax\sin(x)-2\int (ax^2+b)\sin(x)\,dx$

Correct Answers:
- C

Problem 2.
Consider the integral $I = \int_0^4 e^{-x}\,dx$. Let T_n, L_n, and R_n be the approximations to I by the trapezoid rule, the left endpoint rule and the right endpoint rule. Which of the following is true?

- A. $L_n \le T_n \le I \le R_n$
- B. $R_n \le T_n \le I \le L_n$
- C. $L_n \le I \le T_n \le R_n$
- D. $R_n \le I \le T_n \le L_n$
- E. $L_n \le I \le R_n \le T_n$

Correct Answers:
- D

Problem 3.
What substitution should we make to evaluate $\int \sqrt{4-9x^2}\,dx$?

- A. $x = \frac{3}{2}\sin(u)$
- B. $x = \frac{2}{3}\sin(u)$
- C. $x = 2\sin(u)$
- D. $u = \frac{3}{2}\sin(x)$
- E. $u = \frac{2}{3}\sin(x)$

Correct Answers:
- B

Problem 4.
Which is the correct form of the partial fraction expansion of $\frac{x^2+7x-11}{(x^2+4x+7)(x^2-1)(x+1)}$?

- A. $\frac{B}{(x+1)^2}+\frac{C}{x-1}+\frac{Dx+E}{x^2+4x+7}$
- B. $\frac{A}{x+1}+\frac{Bx+C}{x^2-1}+\frac{Dx+E}{x^2+4x+7}$
- C. $\frac{A}{x+1}+\frac{B}{(x+1)^2}+\frac{C}{x-1}+\frac{D}{x^2+4x+7}$
- D. $\frac{A}{x+1}+\frac{B}{(x+1)^2}+\frac{C}{x-1}+\frac{Dx+E}{x^2+4x+7}$
- E. $\frac{A}{x-1}+\frac{B}{(x-1)^2}+\frac{C}{x+1}+\frac{Dx+E}{x^2+4x+7}$

Correct Answers:
- D

Problem 5.
Use the decomposition
$$\frac{x^2-x+6}{x^3+3x} = -\frac{x+1}{x^2+3}+\frac{2}{x}$$
to evaluate the integral
$$\int \frac{x^2-x+6}{x^3+3x}\,dx$$

- A. $-\frac{1}{2}\ln(x^2+3)-\frac{1}{\sqrt{3}}\arctan(x/\sqrt{3})+\ln(x^2)+C$
- B. $\frac{1}{\sqrt{3}}\ln(x^2+3)-\frac{1}{\sqrt{2}}\arctan(x/\sqrt{2})+\ln(x^2)+C$
- C. $\ln(x^2+3)+\frac{1}{\sqrt{3}}\arctan(x/\sqrt{2})+\ln(x^2)+C$
- D. $\frac{1}{\sqrt{3}}\ln(x^2+3)-\arctan(x/\sqrt{2})+\ln(x^2)+C$
- E. $\ln(x^2+3)-\arctan(x/\sqrt{3})+\ln(x^2)+C$

Correct Answers:
- A

Problem 6.
Let $a > 0$ be a fixed number. Evaluate the improper integral $\int_a^\infty x^2 e^{-x^3}\,dx.$

- A. ∞
- B. 0
- C. $-\frac{1}{e^{a^3}}$

- D. $\dfrac{1}{3e^{a^3}}$
- E. e^{a^3}

Correct Answers:

- `D`

Problem 7.

If $x = \sin(u)$ and $-\pi/2 < u < \pi/2$, express $\cot(u)$ in terms of x.

- A. $\dfrac{1}{x}$
- B. $\sqrt{1-x^2}$
- C. $\dfrac{\sqrt{1-x^2}}{x}$
- D. $\dfrac{1}{\sqrt{1-x^2}}$
- E. $\dfrac{x}{\sqrt{1-x^2}}$

Correct Answers:

- `C`

Problem 8.

8. **(5 points)** `local/rmb-problems/lim-seq-num.pg`

Consider the sequence $a_n = \dfrac{5n^2+3n+6}{4n^2+3n-2}$. Find the value of the limit $\lim\limits_{n\to\infty} a_n$.

$\lim\limits_{n\to\infty} a_n =$ ____________

Your answer should be correctly rounded to three decimal places, or more accurate. Exact answers are preferred.

Correct Answers:

- `1.25`

Generated by ©WeBWorK, http://webwork.maa.org, Mathematical Association of America

This is the free response part of Exam 1. There are 3 questions, each worth 20 points. Please write your solutions in full, clearly indicating each step leading to the final answer. Omitting details will result in a lower grade.

10 → **Question 1.** (a) Use an appropriate u-substitution to evaluate

$$\int \frac{e^{-1/x}}{x^2}\,dx.$$

2

SOLUTION: Take $u = -1/x$. Then $du = 1/x^2$ and

$$\int \frac{e^{-1/x}}{x^2}\,dx = \int e^u\,du = e^u + C = e^{-1/x} + C.$$

4 4

- $u = 1/x$ works too
- Ignore forgotten C

10 → (b) Determine whether the improper integral

$$\int_1^\infty \frac{e^{-1/x}}{x^2}\,dx$$

converges and if it does converge, determine its value.

SOLUTION: We have, for $t \geq 1$,

$$\int_1^t \frac{e^{-1/x}}{x^2}\,dx = \left[e^{-1/x}\right]_{x=1}^{x=t} = e^{-1/t} - e^{-1}$$ 5

and

$$\lim_{t\to\infty}\left(e^{-1/t} - e^{-1}\right) = e^0 - e^{-1} = 1 - \frac{1}{e}.$$

Thus

$$\int_1^\infty \frac{e^{-1/x}}{x^2}\,dx$$

converges and its value is $1 - \dfrac{1}{e}$. 5

★

(next page)

20 **Question 2.** Evaluate the integral

$$\int \frac{3x^2+5x+3}{x^3+x}\,dx.$$

SOLUTION: Since $x^3+x = x(x^2+1)$, the partial fraction decomposition of the integrand has the form

$$\frac{3x^2+5x+3}{x^3+x} = \frac{A}{x} + \frac{Bx+C}{x^2+1}. \quad 4$$

Since

$$\frac{A}{x} + \frac{Bx+C}{x^2+1} = \frac{A(x^2+1)+x(Bx+C)}{x^3+x} = \frac{(A+B)x^2+Cx+A}{x^3+x},$$

comparing the coefficients of similar powers of x in the numerators gives

$$A = 3, \quad B = 0, \quad C = 5. \quad \leftarrow 6$$

Thus

$$\frac{3x^2+5x+3}{x^3+x} = \frac{3}{x} + \frac{5}{x^2+1}$$

and

$$\int \frac{3x^2+5x+3}{x^3+x}\,dx = \underbrace{3\ln|x|}_{5} + \underbrace{5\tan^{-1}x}_{5} + C. \quad 10$$

★

- forgotten | | : -1
- ignore forgotten C

20 → **Question 3.** Evaluate

$$\int \frac{\sin^3(x)}{\cos^2(x)}\,dx.$$

4

SOLUTION: Using the substitution $\underline{u = \cos x}$ and

$$du = -(\sin x)\,dx \quad \sin^2(x) = 1 - u^2,$$

• $u = -\cos x$ works too

we have

$$\begin{aligned}
\int \frac{\sin^3(x)}{\cos^2(x)}\,dx &= -\int \frac{1-u^2}{u^2}\,du \\
&= \frac{1}{u} + u + C \\
&= \frac{1}{\cos x} + \cos x + C \\
&= \sec x + \cos x + C.
\end{aligned}$$

6

← 5

5

• either form of answer acceptable ★

• ignore forgotten C

(end of exam questions)