Exam 1

Name: _____ Section: _____

Do not remove this answer page — you will return the whole exam. You will be allowed two hours to complete this test. No books or notes may be used. You may use a graphing calculator during the exam, but NO calculator with a Computer Algebra System (CAS) or a QWERTY keyboard is permitted. Absolutely no cell phone use during the exam is allowed.

The exam consists of 10 multiple choice questions and 5 free response questions. Record your answers to the multiple choice questions on this page by filling in the circle corresponding to the correct answer.

Show <u>all work</u> to receive full credit on the free response problems. It will also help you check your answers to show work on multiple choice problems.

Multiple Choice Questions

1	(A)	(B)	(C)	(D)	(E)	**6**	(A)	(B)	(C)	(D)	(E)
2	(A)	(B)	(C)	(D)	(E)	**7**	(A)	(B)	(C)	(D)	(E)
3	(A)	(B)	(C)	(D)	(E)	**8**	(A)	(B)	(C)	(D)	(E)
4	(A)	(B)	(C)	(D)	(E)	**9**	(A)	(B)	(C)	(D)	(E)
5	(A)	(B)	(C)	(D)	(E)	**10**	(A)	(B)	(C)	(D)	(E)

Multiple Choice	11	12	13	14	15	Total Score
50	10	10	10	10	10	100

Trig Identities

- $\sin^2(x) + \cos^2(x) = 1$ and $\tan^2(x) + 1 = \sec^2(x)$
- $\sin^2(x) = \frac{1}{2}(1 - \cos(2x))$ and $\cos^2(x) = \frac{1}{2}(1 + \cos(2x))$
- $\sin(x + y) = \sin(x)\cos(y) + \cos(x)\sin(y)$ and $\cos(x + y) = \cos(x)\cos(y) - \sin(x)\sin(y)$

Multiple Choice Questions

1. (5 points) Find $\int x^2 \sin(x)\, dx$.

   A. $(2 - x)\cos(x) + 2x\sin(x) + C$
   B. $(2x - x^2)\cos(x) + C$
   C. $(2 - x^2)\cos(x) + 2x\sin(x) + C$
   D. $(2x - x^2)\sin(x) + C$
   E. $(1 + x)\cos(x) + x\sin(x) + C$

2. (5 points) If $f(0) = 4$, $f(1) = 4$, $f'(0) = 1$ and $f'(1) = 2$, and $f''(x)$ is continuous, what is $\int_0^1 xf''(x)\, dx$?

   A. -2
   B. 2
   C. 4
   D. -4
   E. 0

3. (5 points) Find $\int \frac{4}{\sqrt{16-x^2}}\,dx$.

A. $4\arcsin(\frac{x}{4})+C$

B. $4\arccos(\frac{x}{4})+C$

C. $\frac{-2}{16-x^2}+C$

D. $\frac{2}{4-x^2}+C$

E. $\frac{2x}{4-x^2}+C$

4. (5 points) Find $\int_0^{\frac{\pi}{2}} (1+\cos(x))^2\,dx$.

A. $\frac{3\pi+8}{4}$

B. $\frac{3\pi+6}{4}$

C. $\frac{3\pi}{4}$

D. $3\pi+2$

E. $\frac{3\pi}{2}$

5. (5 points) Find $\int \frac{x+1}{x^2+1}\,dx$

A. $\ln|x-1|+C$

B. $\frac{1}{2}\arctan(x)+C$

C. $\ln|x-3|+C$

D. $\frac{1}{2}\ln|x^2+1|+\arctan(x)+C$

E. $\ln|x-1|-\ln|x+1|+C$.

6. (5 points) For what values of p does $\displaystyle\int_1^\infty \frac{1}{x^{\frac{3p}{2}}}\,dx$ converge?

A. $p \geq \frac{2}{3}$

B. $p \geq \frac{3}{2}$

C. $p > \frac{3}{2}$

D. $p \leq \frac{3}{2}$

E. $p > \frac{2}{3}$

7. (5 points) What is the form of the partial fraction decomposition of

$$\frac{3x+1}{(x+1)(x-2)^2(x^2-1)}?$$

A. $\displaystyle\frac{A}{x+1}+\frac{B}{x-2}+\frac{C}{(x-2)^2}+\frac{Dx+E}{x^2-1}$

B. $\displaystyle\frac{A}{x+1}+\frac{Bx+C}{(x-2)^2}+\frac{Dx+E}{x^2-1}$

C. $\displaystyle\frac{A}{x-1}+\frac{B}{(x-1)^2}+\frac{C}{x-2}+\frac{D}{(x-2)^2}+\frac{Ex+F}{x+1}$

D. $\displaystyle\frac{A}{(x-1)^2}+\frac{B}{(x-2)^2}+\frac{C}{x+1}$

E. $\displaystyle\frac{A}{x-1}+\frac{B}{x+1}+\frac{C}{(x+1)^2}+\frac{D}{x-2}+\frac{E}{(x-2)^2}$

8. (5 points) Find the coefficient B in the partial fraction decomposition

$$\frac{x-3}{(x-1)(x+1)(x+2)} = \frac{A}{x-1}+\frac{B}{x+1}+\frac{C}{x+2}$$

A. $B = -2$

B. $B = 2$

C. $B = -4$

D. $B = -1$

E. $B = 1$

9. (5 points) Let $f(x)$ be a function that satisfies $|f''(x)| \leq 6$ on the interval $[1, 3]$. Choose the smallest n so that $E_T = |T_n - \int_1^3 f(x)dx| \leq .0001$.

A. $n = 200$

B. $n = 100$

C. $n = 70$

D. $n = 30$

E. $n = 300$

10. (5 points) Find the trapezoid rule estimate of $\int_1^5 \frac{1}{x}\, dx$ for $n = 4$.

A. $S_4 = \frac{1}{2}$

B. $S_4 = \frac{60}{101}$

C. $S_4 = \frac{73}{45}$

D. $S_4 = \frac{10}{17}$

E. $S_4 = \frac{3}{5}$

Free Response Questions

11. (a) (8 points) Compute $\int xe^{2x}\,dx$

Solution:

(b) (2 points) Compute $\int_0^2 xe^{2x}\,dx$

12. (10 points) Compute $\displaystyle\int \frac{\tan^3(x)}{\cos^7(x)}\,dx$

Solution:

13. (10 points) Compute $\int_0^\infty xe^{-x^2}\,dx$.

Solution:

14. (10 points) Using the method of partial fractions, compute

$$\int \frac{1}{x(x^2+1)}\,dx.$$

Solution:

15. (a) (4 points) Use the midpoint rule to estimate the integral

$$\int_1^7 \frac{1}{x^2}\, dx$$

Use six intervals (ie find M_6).

Solution:

(b) (4 points) Use the trapezoid rule to estimate the integral

$$\int_1^7 \frac{1}{x^2} dx$$

Use six intervals (ie find T_6).

Solution:

(c) (2 points) Which of these estimates is closer to the actual value?

Solution: