Instructions: Each question is followed by space to write your answer. Please write your solutions neatly in the space below the question. *Show your work.* Answers without justification will receive no credit. Partial credit for a problem will be given only when there is coherent written evidence that you have solved part of the problem. In particular, answers that are obtained simply as the output of calculator routines will receive no credit.

This examination consists of 9 questions. Please make sure you have a complete exam. Please note that you are expected to provide supporting calculations for, and fully justify how you arrived at your answers. Unsupported answers will receive no credit.

Name: _____

Section: _____

Last four digits of student identification number: _____

Some derivatives

$$\frac{d}{dx}\tan x = \sec^2 x$$

$$\frac{d}{dx}\sec x = \sec x \tan x$$

$$\frac{d}{dx}\arcsin x = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\arctan x = \frac{1}{1+x^2}$$

$$\frac{d}{dx}\ln(\sec x + \tan x) = \sec x$$

| Problem	Score	Total
1		10
2		5
3		15
4		10
5		10
6		14
7		14
8		17
9		15
Total		110

1) (10 pts) Suppose $\sum_{n=0}^{\infty} a_n\, x^n$ is the Taylor series for $f(x) = e^{-x/3}$, expanded about $x = 0$.

(a) (5 pts) Calculate a general expression (a formula) expressing a_n in terms of n.

(b) (5 pts) Calculate $T_3(x) = \sum_{n=0}^{3} a_n\, x^n$, the degree three Taylor polynomial for $f(x)$ expanded about the origin.

2) (5 pts) Calculate the Taylor series for $f(x) = x^2 - 5x + 6$ expanded about $x = 2$.

3)   (15 pts) The questions in this problem refer to R, the shaded region in the diagram, which is bounded by the graphs of $f(x) = x^2 - 3x + 3$ and $g(x) = x$.
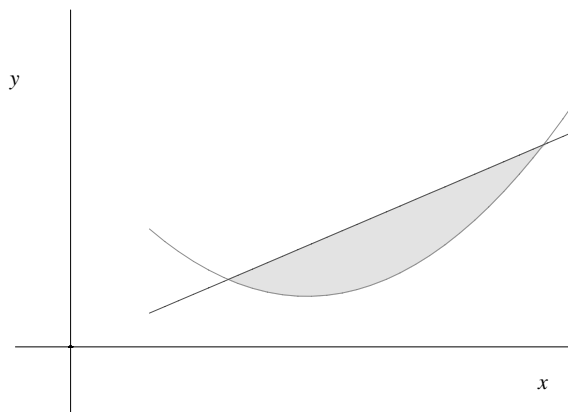
(a) (5 pts) Express the area of R as a definite integral with explicit integrand and limits of integration. You do not need to evaluate the integral.

(b) (5 pts) Calculate the volume of the solid of revolution generated by rotating R about the x-axis. Express the volume as a definite integral with explicit integrand and limits of integration. You do not need to simplify the integrand or evaluate the integral.

(c) (5 pts) Calculate the volume of the solid of revolution generated by rotating R about the (horizontal) line $y = -1$. Express the volume as a definite integral with explicit integrand and limits of integration. You do not need to simplify the integrand or evaluate the integral.

4)   (10 pts) S is the shaded region in the diagram below that is bounded by the graphs of $f(x) = 2x - x^2$, $g(x) = x$, and the line $x = 2$. Calculate the area of S.
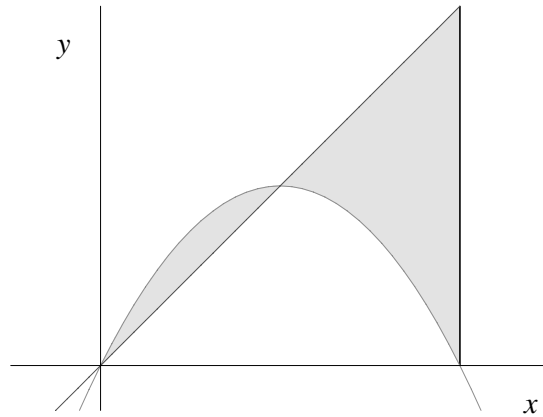
5) (10 pts) Recall that Hooke's Law says that the force, F, required to expand or compress a spring to a length x beyond its natural length is proportional to the length by which it is expanded or compressed. That is, $F = K x$ where K is a constant dependent on the spring. Recall also that if F is measured in newtons and x is measured in meters, then the unit for work is a newton-meter, which is called a joule. Suppose that it takes a force of 100 newtons to compress a certain spring to a length of 4 centimeters less than its natural length. How much work in joules must be done to compress the spring to a length of 10 centimeters less than its natural length?

6)   (14 pts) Calculate the following integrals.

(a) (7 pts) $\displaystyle\int \frac{\sin(3\,x)}{1 + \cos^2(3\,x)}\,dx.$

(b) (7 pts) $\displaystyle\int_1^2 \frac{1}{x\,(1 + \ln(x))^2}\,dx.$

7)   (14 pts) Calculate the following integrals.

(a) (7 pts) $\int \sin^2(x)\,dx$.

(b) (7 pts) $\int \sec^4(x)\,dx$.

8)   (17 pts) Calculate the following integrals.

(a) (7 pts) $\displaystyle\int \frac{1}{\sqrt{4 - x^2}}\,dx.$

(b) (10 pts) $\displaystyle\int \frac{1}{\sqrt{4 + x^2}}\,dx.$

9) (15 pts) Calculate the following integrals.

(a) (7 pts) $\int x \, \cos(x) \, dx$.

(b) (8 pts) $\int x \, \ln(x) \, dx$.