
Problem 1.
5. (5 points) `local/rmb-problems/integral-test-num.pg`

Give the whole number N for which we have $\int_{N+1}^{\infty} \frac{1}{x^2}\,dx \le \sum_{k=6}^{\infty} \frac{1}{k^2} \le \int_{N}^{\infty} \frac{1}{x^2}\,dx$.

$N =$ ______________________ .

Evaluate one of the integrals above to find A so that $\sum_{k=6}^{\infty} \frac{1}{k^2} \le A$

$A =$ ______________________.

The answers must be correctly rounded to four decimal places or more accurate.

Problem 2.
7. (5 points) `local/rmb-problems/power-series-num.pg`

Consider the function

$$F(x) = \int_0^x \frac{1}{1+t^3}\,dt.$$

The first three (non-zero) terms of the Mclaurin series for F are $Ax + Bx^4 + Cx^7$. Give the values of A, B, and C.
$A =$ ______________________, $B =$ ______________________, $C =$ ______________________
Your answers must be correctly rounded to four decimal places, or more accurate. Exact answers are preferred.

Problem 3.
1. (5 points) `local/rmb-problems/geom-series2-num.pg`

Find the sum of the geometric series $\sum_{k=3}^{\infty} 11 \cdot 3^{-k} =$ ________
Your answer should be correctly rounded to four decimal places, or more accurate.

Problem 4.
3. (5 points) `local/rmb-problems/alt-series-div-mc.pg`

Select the one correct statement for the series $\sum_{k=1}^{\infty} \frac{(-1)^k k}{4k+7}$?

- A. The series is absolutely convergent.
- B. The series is conditionally convergent.
- C. The series is absolutely convergent, but not convergent.
- D. The series is divergent.
- E. None of the above

Problem 5.
2. (5 points) `local/rmb-problems/telescope-num.pg`

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Find the value of the finite sum $\sum_{k=5}^{25} \left(\frac{7}{k} - \frac{7}{k+1} \right) =$ ________
Your answer should be correctly rounded to four decimal places, or more accurate.

Problem 6.
4. (5 points) `local/rmb-problems/limit-comp-mc.pg`

Consider the series $\sum_{j=1}^{\infty} \frac{1}{3^j + 17j + 2}$ and select the correct statement.

- A. Using the comparison test to compare with the series $\sum_{j=1}^{\infty} \frac{1}{3^j}$ establishes of the divergence of the series.
- B. Using the comparison test to compare with the series $\sum_{j=1}^{\infty} \frac{1}{3^j}$ establishes convergence of the series.
- C. The series converges conditionally, but we cannot use a comparison test to establish conditional convergence.
- D. Using the comparison test to compare with the series $\sum_{j=1}^{\infty} \frac{1}{17j}$ establishes divergence of the series.
- E. Using the comparison test to compare with the series $\sum_{j=1}^{\infty} \frac{1}{17j}$ establishes convergence of the series.

Problem 7.
6. (5 points) `local/rmb-problems/ratio-test-num.pg`

Consider the power series $\sum_{k=1}^{\infty} \frac{5^k x^k}{k!}$ with terms $a_k = \frac{5^k x^k}{k!}$.
Find the limit of the ratio $\lim_{k\to\infty} \frac{a_{k+1}}{a_k} =$ ______________________

Give the radius of convergence of the series $\sum_{k=1}^{\infty} \frac{5^k x^k}{k!}$.

- A. ∞
- B. $1/5$

- C. 0
- D. 1
- E. 5

Problem 8.

8. **(5 points)** `local/rmb-problems/cond-abs-conv-mc.pg`

Determine if each series converges absolutely, converges conditionally, or diverges.

$$\sum_{k=1}^{\infty} \frac{(-1)^k}{k^2}$$

- A. The series diverges.
- B. The series converges conditionally.
- C. The series converges absolutely.

$$\sum_{k=1}^{\infty} (-1)^k 2^k$$

- A. The series converges absolutely.
- B. The series converges conditionally.
- C. The series diverges.

This is the free response part of Exam 2. There are 3 questions, each worth 20 points. Please write your solutions in full, clearly indicating each step leading to the final answer. Omitting details will result in a lower grade.

Question 1. Determine the **radius** and the **interval** of convergence of the power series

$$\sum_{n=0}^{\infty} \frac{(-1)^n(2n+3)(5n+11)x^n}{7^n}.$$

Be sure to test the endpoints, and clearly label your final answers.

(next page)

Question 2. For each of the following three series, use the limit comparison test to determine if the series converges.

(a) $$\sum_{k=1}^{\infty} \frac{1}{3^k} \left(\frac{4k^2+5}{2k^2+1} \right)$$

(b) $$\sum_{k=1}^{\infty} \left(\frac{k^2+2k-1}{3k^3+5k} \right)$$

(third series on next page)

(c) $$\sum_{k=1}^{\infty}\left(\frac{2k^3+3k^2}{5k^7+7k^5+12}\right)$$

(next page)

Question 3. Let $f(x) = \ln(3 - 2x)$.

(a) Find the derivative $f'(x)$ of $f(x)$, and then find a power series centered at 0 for $f'(x)$. Give the radius of convergence for the power series.

(b) Use your answer from part (a) to find a power series centered at 0 for $f(x)$. Find the radius of convergence for the power series.

(end of exam questions)