MA 114 — Calculus II Spring 2010
Exam 3 April 13, 2010

Instructions: This examination consists of 9 questions on 10 pages. Please make sure you have a complete exam. This is a closed book exam. No books or notes are to be used during the exam. You may use a graphing calculator if it does not have symbolic manipulation capabilities. However, any device capable of electronic communication (cell phone, pager, etc.) must be turned off and out of sight during the exam.

Each question is followed by space to write your answer. Please write your solutions neatly in the space below the question. *Show your work.* Answers without justification will receive no credit. Partial credit for a problem will be given only when there is coherent written evidence that you have solved part of the problem. In particular, answers that are obtained simply as the output of calculator routines will receive no credit.

Name: ______________________________

Section: __________

Last four digits of student identification number: __________

If $|f''(x)| \le K$ for $a \le x \le b$, then

$$|E_T| \le \frac{K(b-a)^3}{12n^2} \text{ and}$$

$$|E_M| \le \frac{K(b-a)^3}{24n^2}.$$

If $|f^{(4)}(x)| \le K$ for $a \le x \le b$, then

$$|E_S| \le \frac{K(b-a)^5}{180n^4}.$$

Problem	Score	Total
1		10
2		15
3		10
4		16
5		9
6		20
7		15
8		5
9		10
Total		110

1) (10 pts)

(a) (5 pts) Find $\dfrac{d}{dx}\ln(\tanh(x))$.

(b) (5 pts) Find $\displaystyle\int \tanh(2x)\,dx$.

2) (15 pts) This problem considers the curve $y = \frac{1}{3}x^{3/2}$, $0 \leq x \leq 12$.

(a) (12 pts) Find the exact length of the curve.

(b) (3 pts) Find the distance between the endpoints of the curve.

3) (10 pts) The following are some of the values for an unknown function $G(x)$.

x	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
$G(x)$	0.000	0.100	0.198	0.296	0.390	0.480	0.567	0.649	0.726	0.798	0.866

Let $I = \displaystyle\int_0^1 G(x)\,dx$.

(a) (5 pts) Use the trapezoidal rule with $n = 5$ to obtain an approximate value of I.

(b) (5 pts) Use the midpoint rule with $n = 5$ to obtain an approximate the value of I.

4) (16 pts) Let $f(x)$ be defined in pieces by $f(x) = \begin{cases} 2+x & \text{when } -2 \le x \le 0 \\ x^{-1/2} & \text{when } 0 < x \le 1 \\ x^{-2} & \text{when } 1 \le x \end{cases}$.

(a) (3 pts) Is the integral $\int_{-2}^{1} f(x)\,dx$ an improper integral? Explain why or why not?

(b) (5 pts) If it is an improper integral does it converge? If so, to what value? If it is not an improper integral, what is its value?

(c) (5 pts) Does $\int_{1}^{\infty} f(x)\,dx$ converge or diverge? If it diverges, why? If it converges, to what value does it converge?

(d) (3 pts) Does $\int_{-2}^{\infty} f(x)\,dx$ converge or diverge? Explain why or why not.

5) (9 pts) Use the inequality

$$0 \leq s - s_n \leq \int_n^{\infty} f(x)\,dx$$

from the section on the integral test to determine the least number of terms of the series

$$\sum_{n=1}^{\infty} \frac{1}{n^3}$$

that should be added (beginning with the first) to obtain the sum of the series to within 0.01. Your solution should give a choice for $f(x)$ and should verify that it satisfies the hypothesis of the given inequality.

6) (20 pts) This problem considers the integral $I = \displaystyle\int_0^2 \sin(2x)\,dx$.

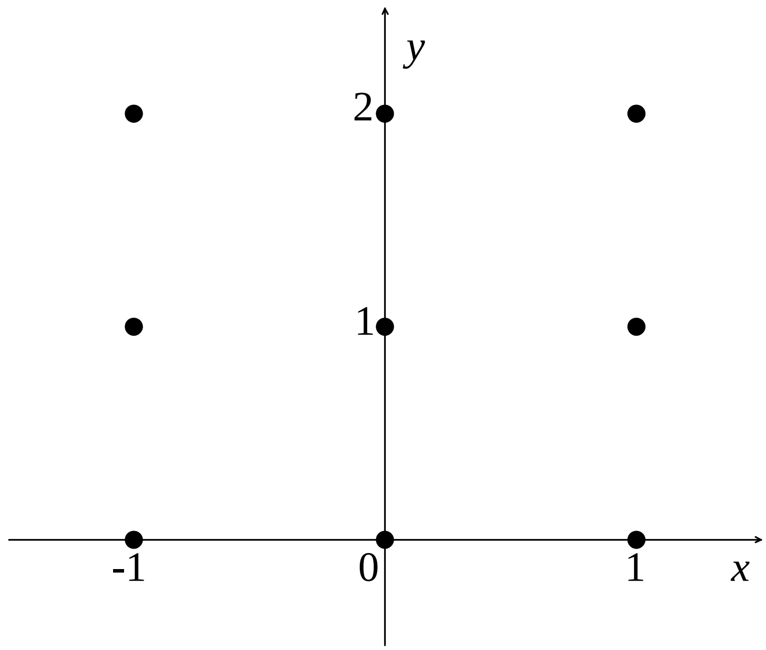
(a) (3 pts) Give the smallest value of K in the error bound for the Midpoint Rule (given on the front page of the exam).

(b) (9 pts) Use the error bound for the Midpoint Rule to find the least value of n which guarantees that the Midpoint Rule obtains a value within 0.01 of the actual value of I.

(c) (8 pts) Use Simpson's Rule with $n = 4$ to approximate I. (Remember to set your calculator for radians.)

7) (15 pts) Consider the differential equation $\frac{dy}{dx} = -xy^2$. Let $y = f(x)$ be the particular solution to this differential equation with the initial condition $f(-1) = 1$.

(a) (4 pts) On the axes provided, sketch the direction field for the given differential equation at the nine points indicated.



(b) (3 pts) Use your direction field or other information you have found to sketch the solution curve on the above graph that passes through the point $(-1, 1)$.

(c) (8 pts) Find the solution $y = f(x)$ to the given differential equation with the initial condition $f(-1) = 1$.

8) (5 pts) Use Euler's method to approximate the value of y at $x = 1$ on the solution curve to the differential equation

$$\frac{dy}{dx} = (x + y)y,$$

that passes through the point $(0, 2)$. Use stepsize $h = \frac{1}{2}$ and use 2 steps. Fill in the table below and show all of your calculations.

Step	x_n	y_n
0	0	2
1	$\frac{1}{2}$	
2		

9) (10 pts) Find $\displaystyle\int \frac{3x^2+x+1}{x(x^2+1)}\,dx$.