Exam 4

Name: _____   Section: _____

Do not remove this answer page — you will return the whole exam. You will be allowed two hours to complete this test. No books or notes may be used. You may use a graphing calculator during the exam, but NO calculator with a Computer Algebra System (CAS) or a QWERTY keyboard is permitted. Absolutely no cell phone use during the exam is allowed.

The exam consists of 10 multiple choice questions and 5 free response questions. Record your answers to the multiple choice questions on this page by filling in the circle corresponding to the correct answer.

Show all work to receive full credit on the free response problems. It will also help you check your answers to show work on multiple choice problems.

Multiple Choice Questions

1 (A) (B) (C) (D) (E)   6 (A) (B) (C) (D) (E)

2 (A) (B) (C) (D) (E)   7 (A) (B) (C) (D) (E)

3 (A) (B) (C) (D) (E)   8 (A) (B) (C) (D) (E)

4 (A) (B) (C) (D) (E)   9 (A) (B) (C) (D) (E)

5 (A) (B) (C) (D) (E)   10 (A) (B) (C) (D) (E)

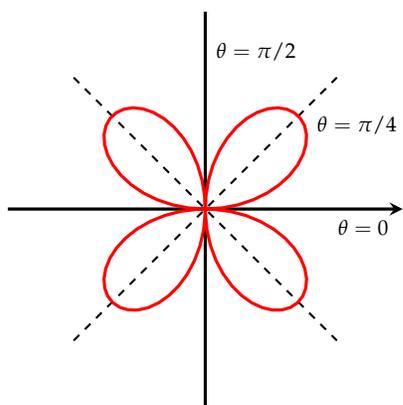| Multiple Choice	11	12	13	14	15	Total Score
50	10	10	10	10	10	100

Multiple Choice Questions

1. (5 points) Identify the type of conic section given by

$$4x^2 = y^2 + 4$$

and give its vertices and foci.

    A. Ellipse, vertices $(\pm 1, 0)$ and $(0, \pm 2)$, foci $(\pm\sqrt{3}, 0)$

    B. Hyperbola, vertices $(\pm 2, 0)$, foci $(\sqrt{5}/2, 0)$

    C. Hyperbola, vertices $(\pm 1, 0)$, foci $(\pm\sqrt{5}/2, 0)$

    D. Hyperbola, vertices $(\pm 1, 0)$, foci $(\pm\sqrt{5}, 0)$

    E. Ellipse, vertices $(\pm 2, 0)$ and $(0, \pm 1)$, foci $(0, \pm\sqrt{3})$

2. (5 points) Which of the following correctly computes the area of one leaf of the curve $r = \sin(2\theta)$ shown below?
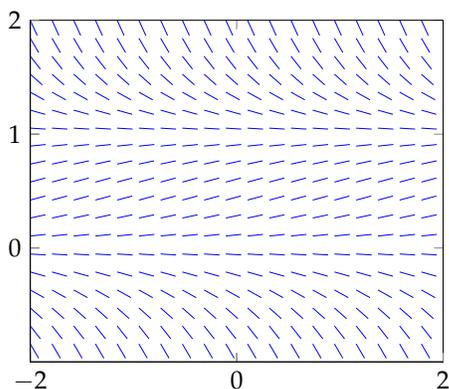

    A. $\displaystyle\int_0^{\pi/2} \frac{1}{2}\sin(2\theta)\,d\theta$

    B. $\displaystyle\int_0^{\pi/2} \frac{1}{2}\sin^2(2\theta)\,d\theta$

    C. $\displaystyle\int_0^{\pi/4} \frac{1}{2}\sin(2\theta)\,d\theta$

    D. $\displaystyle\int_0^{\pi/4} \frac{1}{2}\sin^2(2\theta)\,d\theta$

    E. $\displaystyle\int_0^{\pi} \frac{1}{2}\sin^2(2\theta)\,d\theta$

3. (5 points) Which of the following differential equations has *both* $y = e^{-x}$ and $y = e^{2x}$ as solutions?

    A. $y'' - y' - 2y = 0$

    B. $y'' + y' - 2y = 0$

    C. $y'' = 4y$

    D. $y'' + y = 0$

    E. $y' = y$

4. (5 points) Which of the following equations has the direction field shown?

    A. $y' = y$

    B. $y' = \sin(y)$

    C. $y' = xy$

    D. $y' = y(1 - y)$

    E. $y' = y(2 - y)$

5. (5 points) Which of the following is equal to $\int_0^x t f'(t)\, dt$?

    A. $f(x) - \int_0^x t f(t)\, dt$

    B. $x f(x) - \int_0^x f(t)\, dt$

    C. $x f'(x) - \int_0^x f(t)\, dt$

    D. $f'(x) - \int_0^x t f(t)\, dt$

    E. $f'(x) - \int_0^x t f'(t)\, dt$

6. (5 points) The curve

$$x^2 - 2x + 2y^2 - 8y + 7 = 0$$

is correctly described by which of the following?

     A. Ellipse, center $(2, 1)$, $a = 2$, $b = 1$

     B. Circle, center $(1, 2)$, radius $\sqrt{2}$

     C. Ellipse, center $(1, 2)$, $a = \sqrt{2}$, $b = 1$

     D. Hyperbola, center $(1, 2)$, foci $(1/2, 2)$ and $(3/2, 2)$

     E. Ellipse, center $(2, 1)$, $a = \sqrt{2}$, $b = 1$

7. (5 points) State the partial fraction decomposition of

$$\frac{x^3}{(x^2 + 2x - 3)(x^2 + x + 1)}.$$

     A. $\dfrac{A}{x + 1} + \dfrac{B}{x - 3} + \dfrac{Cx + D}{x^2 + x + 1}$

     B. $\dfrac{A}{x - 1} + \dfrac{B}{x + 3} + \dfrac{Cx + D}{x^2 + x + 1}$

     C. $\dfrac{A}{x - 1} + \dfrac{B}{x + 3} + \dfrac{C}{x^2 + x + 1}$

     D. $\dfrac{A}{x + 1} + \dfrac{B}{x - 3} + \dfrac{C}{x^2 + x + 1}$

     E. $\dfrac{A}{x - 1} + \dfrac{B}{x - 3} + \dfrac{Cx + D}{x^2 + x + 1} + \dfrac{Ex + F}{(x^2 + x + 1)^2}$

8. (5 points) A function has Maclaurin series (about $a = 0$)

$$f(x) = x - \frac{x^3}{3} + \frac{x^5}{5} - \cdots .$$

What is $f'''(0)$?

     A. 0

     B. 2

     C. -2

     D. $-\dfrac{1}{3}$

     E. $\dfrac{1}{3}$

9. (5 points) The substitution $x = 2\sin\theta$ in the integral

$$\int \frac{dx}{x^2\sqrt{4-x^2}}$$

leads to which one of the following?

A. $-\displaystyle\int \frac{d\theta}{\cos^2\theta}$

B. $\displaystyle\int \frac{\cot\theta\, d\theta}{2\sin^2\theta}$

C. $\displaystyle\int \frac{d\theta}{\sin^2\theta}$

D. $\displaystyle\int \frac{d\theta}{4\sin^2\theta}$

E. $-\displaystyle\int \frac{4\, d\theta}{\sin^2\theta}$

10. (5 points) The Taylor series

$$f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n (x-2)^n}{3^n(n+1)}$$

has what radius and interval of convergence?

    A. Radius of convergence 3 and interval of convergence $(-1,5)$

    B. Radius of convergence 3 and interval of convergence $(-2,4)$

    C. Radius of convergence 3 and interval of convergence $(-1,5]$

    D. Radius of convergence 3 and interval of convergence $[-2,4]$

    E. Radius of convergence 3 and interval of convergence $[-1,5)$

Free Response Questions

11. (10 points) Find the equation of the tangent line to the polar curve $r = 2\cos\theta$ at $\theta = \pi/3$.

12. (a) (5 points) Let $\mathscr{R}$ be the region bounded by the curves $y = x^3$ and $y = x$ for $x \geq 0$. Set up but do not evaluate an integral which gives the volume of the solid obtained by rotating $\mathscr{R}$ about the x-axis. Be sure to state which method (disc, washer, shell) you are using. Be sure to state the radius and height of the shell (shell method) or the inner and outer radius of the washer (washer method).
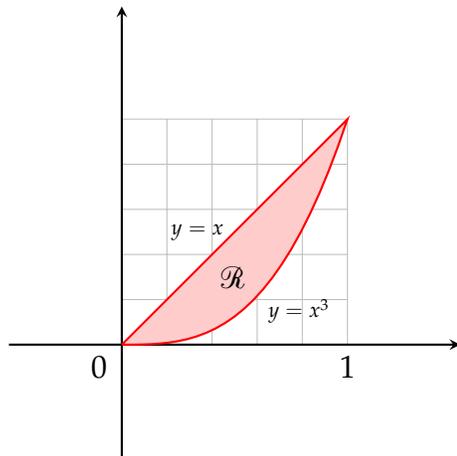
(b) (5 points) Set up but do not evaluate an integral which gives the volume of the solid obtained by rotating the region $\mathscr{R}$ from part (a) about the y-axis. Be sure to state which method (disc, washer, shell) you are using. Be sure to state the radius and height of the shell (shell method) or the inner and outer radius of the washer (washer method).

13. (a) (5 points) Use Euler's method with stepsize 0.25 to estimate $y(1)$ if $y(x)$ solves the initial value problem

$$y' = x - y, \quad y(0) = 1.$$

Show your steps in tabular form, and give results correct to three decimal places.

n	x_n	y_n	$x_n - y_n$
0	0.000	1.000	
1			
2			
3			
4			

(b) (5 points) Using the method of separation of variables, find the general solution of the differential equation

$$\frac{dy}{dx} = 3x^2 y^2$$

Then find the unique solution y with $y(0) = 1/2$.

14. (10 points) Using the method of partial fractions, compute

$$\int_0^1 \frac{x-4}{x^2-5x+6} \, dx.$$

15. (a) (5 points) State the ratio test for a series $\sum_{n=1}^{\infty} a_n$. Be sure to include each of the three cases.

(b) (5 points) Use the ratio test to determine whether the series

$$\sum_{n=1}^{\infty} \frac{9^n}{(n+1)4^{2n+1}}$$

is convergent or divergent.