Name: ______________________ Section: ____________

Do not remove this answer page — you will return the whole exam. You will be allowed two hours to complete this test. No books or notes may be used. You may use a graphing calculator during the exam, but you may not use a calculator that has symbolic manipulation capabilities of any sort. This forbids the use of TI-89, TI-Nspire CAS, HP 48, TI 92, and many others, as stated on the syllabus. Absolutely no cell phone use during the exam is allowed.

The exam consists of 10 multiple choice questions and 5 free response questions. Record your answers to the multiple choice questions on this page by filling in the circle corresponding to the correct answer.

Show all work to receive full credit on the free response problems. You should also show your work on the multiple choice questions as it will make it easier for you to check your work. You should give exact answers, rather than a decimal approximation unless the problem asks for a decimal answer. Thus, if the answer is 2π, you should not give a decimal approximation such as 6.283 as your final answer.

Multiple Choice Questions

1 A B C D E **6** A B C D E

2 A B C D E **7** A B C D E

3 A B C D E **8** A B C D E

4 A B C D E **9** A B C D E

5 A B C D E **10** A B C D E

Multiple Choice	11	12	13	14	15	Total Score
50	10	10	10	10	10	100

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Multiple Choice Questions

1. (5 points) Assuming that $f(x)$ has a continuous second derivative, select the expression that is equal to

$$\int f(x)f''(x)\,dx$$

A. $f(x)f'(x) + \int f'(x)^2\,dx$

B. $f'(x)^2 - \int f(x)^2\,dx$

C. $f(x)f'(x) - \int f'(x)^2\,dx$

D. $f(x)^2 - \int f'(x)^2\,dx$

E. $f'(x)^2 - \int f(x)^2\,dx$

2. (5 points) Choose the proper partial fraction decomposition of

$$f(x) = \frac{x^3+2}{x^2(x-1)(x^2-4)}.$$

[Do not attempt to compute the constants A, B, C, etc.]

A. $\frac{A}{x} + \frac{B}{x-1} + \frac{C}{x^2-4}$

B. $\frac{A}{x^2} + \frac{B}{x-1} + \frac{Cx+D}{x^2-4}$

C. $\frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-1} + \frac{D}{x-2} + \frac{E}{x+2}$

D. $\frac{A}{x^2} + \frac{B}{x-1} + \frac{Cx+D}{x-2} + \frac{Ex+F}{x+2}$

E. $\frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-1} + \frac{Dx+E}{x^2-4}$

3. (5 points) If

$$\sum_{n=k}^{\infty} \frac{1}{3^n} = \frac{1}{54},$$

what is k?

A. 1

B. 2

C. 3

D. 4

E. 5

4. (5 points) Only one of the following series is conditionally convergent. Which one?

A. $\displaystyle\sum_{n=3}^{\infty} (-1)^n \ln(n)$

B. $\displaystyle\sum_{n=3}^{\infty} \ln\left(\frac{1}{n}\right)$

C. $\displaystyle\sum_{n=3}^{\infty} (-1)^n \ln\left(\frac{1}{n}\right)$

D. $\displaystyle\sum_{n=3}^{\infty} \frac{1}{\ln(n)}$

E. $\displaystyle\sum_{n=3}^{\infty} (-1)^n \frac{1}{\ln(n)}$

5. (5 points) The surface S is obtained by rotating the graph of $f(x) = e^{3x}$, $-1 \leq x \leq 1$, about the x-axis. Choose the integral that gives the area of S.

A. $2\pi \int_0^{2\pi} e^{3x}\sqrt{1+9e^{3x}}\,dx$

B. $\pi \int_{-1}^{1} e^{3x}\sqrt{1+9e^{6x}}\,dx$

C. $2\pi \int_{-1}^{1} e^{3x}\sqrt{1+9e^{3x}}\,dx$

D. $2\pi \int_{-1}^{1} \sqrt{1+9e^{3x}}\,dx$

E. $2\pi \int_{-1}^{1} e^{3x}\sqrt{1+9e^{6x}}\,dx$

6. (5 points) Consider the curve with parametric equations $x(t) = t^3$ and $y(t) = \ln(t)$. Find the slope of the tangent to the curve at the point $(1, 0)$.

A. 1/3

B. 2/3

C. $-2/3$

D. 3

E. 3/2

7. (5 points) Which of the following parametric curves is **not** a line segment?

A. $x = 2t^2 + 1, \quad y = 3t^2 + 5, \quad 0 \leq t \leq 1$

B. $x = \cos^2(t), \quad y = 2\sin^2(t), \quad 0 \leq t \leq 1$

C. $x = \tan^2(t), \quad y = \sec^2(t), \quad 0 \leq t \leq 1$

D. $x = 2t + 1, \quad y = 3t^2 + 5, \quad 0 \leq t \leq 1$

E. $x = \ln(t), \quad y = \ln(3t), \quad 2 \leq t \leq 3$

8. (5 points) Find the foci of the hyperbola

$$\frac{y^2}{7} - \frac{x^2}{5} = 1.$$

A. $(\pm 2\sqrt{3}, 0)$

B. $(0, \pm 2\sqrt{3})$

C. $(0, \pm\sqrt{74})$

D. $(0, \pm 2\sqrt{6})$

E. $(\pm 2\sqrt{6}, 0)$

9. (5 points) Find all the values of r for which $y = e^{rx}$ is a solution of the differential equation $y'' + 4y' = 0$.

   A. $r = 0$

   B. $r = 4$

   C. $r = 2$ and $r = -2$

   D. $r = 4$ and $r = -4$

   E. $r = 0$ **and** $r = -4$

10. (5 points) Which of the following substitutions would be most useful to evaluate the integral

$$\int (4 + 9x^2)^{-3/2}\, dx?$$

   A. $x = \frac{3}{2}\tan(\theta),\ (-\pi/2 < \theta < \pi/2)$

   B. $x = \frac{2}{3}\sin(\theta),\ (-\pi/2 \leq \theta \leq \pi/2)$

   C. $x = \frac{2}{3}\sec^2(\theta),\ (-\pi/2 < \theta < \pi/2)$

   D. $x = \frac{3}{2}\sec(\theta),\ (-\pi/2 < \theta < \pi/2)$

   E. $x = \frac{2}{3}\tan(\theta),\ (-\pi/2 < \theta < \pi/2)$

Free Response Questions

11. Let R be the region enclosed by the curves $y = e^{-x^2}$, $y = 0$, $x = 0$, $x = 1$. We form a solid of revolution S by rotating the region R about the y-axis.

   (a) (6 points) Use the method of cylindrical shells to write an integral giving the volume V of the solid S.

   Solution: Since R is the region below the graph of $f(x) = e^{-x^2}$ for $0 \le x \le 1$, we have
   $$V = \int_0^1 2\pi x f(x)\,dx = \int_0^1 2\pi x e^{-x^2}\,dx.$$

   (b) (4 points) Evaluate the integral to find the volume of S.

   Solution: To evaluate the above integral, we let $u = x^2$, and $du = 2x\,dx$. Then
   $$V = \int_0^1 \pi e^{-u}\,du = \pi \left[-e^{-x^2}\right]_{x=0}^{x=1} = \pi\left(1 - \frac{1}{e}\right).$$

12. (a) (5 points) Use the identity

$$\frac{x}{(1-2x)(1-x)} = \frac{1}{1-2x} - \frac{1}{1-x}$$

and the formula for the geometric series to find the first 5 terms of the Maclaurin series for

$$f(x) = \frac{x}{(1-2x)(1-x)}.$$

Solution: We have

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + x^4 + \cdots = \sum_{n=0}^{\infty} x^n,$$

and

$$\frac{1}{1-2x} = 1 + 2x + 4x^2 + 8x^3 + 16x^4 + \cdots = \sum_{n=0}^{\infty} 2^n x^n.$$

Thus

$$\frac{x}{(1-2x)(1-x)} = 0 + x + 3x^2 + 7x^3 + 15x^4 + \cdots = \sum_{n=0}^{\infty} (2^n - 1)x^n.$$

(b) (5 points) Use the Ratio Test to find the radius of convergence of the series

$$\sum_{n=0}^{\infty} (3^n - 1)x^n.$$

Solution: We have

$$\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n\to\infty} \left| \left(\frac{3^{n+1} - 1}{3^n - 1} \right) x \right| = 3\,|x|\,.$$

Thus the series converges for $|x| < 1/3$ and diverges for $|x| > 1/3$. Therefore the radius of convergence is $1/3$.

13. (10 points) Find the area A of the region bounded by the polar curve

$$r = \sin\theta + \cos\theta, \quad 0 \leq \theta \leq \pi.$$

Solution: We have

$$A = \frac{1}{2}\int_0^{\pi} (\sin\theta + \cos\theta)^2 \, d\theta = \frac{1}{2}\int_0^{\pi} (1 + 2\sin\theta\cos\theta) \, d\theta = \frac{1}{2}\int_0^{\pi} (1 + \sin(2\theta)) \, d\theta = \frac{\pi}{2}.$$

A different, equally acceptable, solution : Since

$$r^2 = r\cos\theta + r\sin\theta,$$

we have

$$x^2 + y^2 = x + y,$$

and completing the squares gives

$$\left(x - \frac{1}{2}\right)^2 + \left(y - \frac{1}{2}\right)^2 = \frac{1}{2}.$$

Thus the region is bounded by a circle of radius $1/\sqrt{2}$ and has area $\pi/2$.

14. Consider the ellipse with equation $9x^2 - 18x + 4y^2 = 27$.

(a) (4 points) Complete the square to put the equation in the form

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1.$$

Solution: Adding 9 to both sides of the equation we have

$$9x^2 - 18x + 9 + 4y^2 = 9(x-1)^2 + 4y^2 = 36.$$

Dividing by 36 gives

$$\frac{(x-1)^2}{4} + \frac{y^2}{9} = 1.$$

(b) (6 points) Give the endpoints of the major and minor axes of the ellipse and the foci of the ellipse.

Solution: The ellipse

$$\frac{(x-1)^2}{4} + \frac{y^2}{9} = 1$$

is obtained from the ellipse

$$\frac{x^2}{4} + \frac{y^2}{9} = 1$$

by shifting it 1 unit to the right. Since the endpoints of the major and minor axes and the foci of the ellipse

$$\frac{x^2}{4} + \frac{y^2}{9} = 1$$

are respectively $(0, \pm 3)$, $(\pm 2, 0)$, and $(0, \pm\sqrt{5})$, the endpoints of the major and minor axes and the foci of the ellipse

$$\frac{(x-1)^2}{4} + \frac{y^2}{9} = 1$$

are $(1, \pm 3)$ (major axis), $(-1, 0)$ and $(3, 0)$ (minor axis), and $(1, \pm\sqrt{5})$ (foci).

15. (10 points) Find the solution to the initial value problem

$$y' = 2xe^{-y}, \qquad y(0) = 0.$$

Solution: This is a separable equation. Separating the variables, we have

$$e^y \, dy = 2x \, dx.$$

Integration of both sides gives

$$e^y = x^2 + C.$$

From the initial condition $y(0) = 0$, we see that $1 = 0 + C$, i.e. $C = 1$ and

$$e^y = x^2 + 1.$$

Solving for y gives

$$y = \ln(x^2 + 1).$$