MA 114 Worksheet # 20: Arc Length and Surface Area

1. Conceptual Understanding:
   (a) Write down the formula for the arc length of a function $f(x)$ over the interval $[a, b]$ including the required conditions on $f(x)$.
   (b) Write down the formula for the (surface) area of the surface obtained by rotating the graph of $f(x)$ about the x-axis for $a \leq x \leq b$. How would this formula change if the graph were instead rotated about $y = c$?

2. Find an integral expression for the arc length of the following curves. Do ***not*** evaluate the integrals.
   (a) $f(x) = \sin(x)$ from $x = 0$ to $x = 2$.
   (b) $f(x) = x^4$ from $x = 2$ to $x = 6$.
   (c) $x^2 + y^2 = 1$

3. Find the arc length of the following curves.
   (a) $f(x) = x^{3/2}$ from $x = 0$ to $x = 2$.
   (b) $f(x) = \ln(\cos(x))$ from $x = 0$ to $x = \pi/3$.

4. Set up a function $s(t)$ that gives the arc length of the curve $f(x) = 2x + 1$ from $x = 0$ to $x = t$. Find $s(4)$.

5. Calculate the arc length of $f(x) = x^2$ over $[0, 1]$. [Hint: You will need to use a trigonometric substitution.]

6. Calculate the arc length of the graph of $f(x) = mx + r$ over $[a, b]$ in two ways: using the Pythagorean Theorem and using the arc length integral. [Hint: Make the arc of $f(x) = mx + r$ from $[a, b]$ the hypotenuse of a right triangle with legs $(b - a)$ and $m(b - a)$.]

7. Use Simpson's Rule with $n = 6$ to approximate the arc length of $f(x) = \sin(x)$ over $[0, \pi]$.

   For Problems 8–10, compute the surface area for a revolution about the x-axis over the given interval.

8. $y = x$, $[0, 4]$

9. $y = x^3$, $[0, 2]$

10. $y = \left(4 - x^{2/3}\right)^{3/2}$, $[0, 8]$