MA 114 Worksheet # 27:
Tangent lines to parametric equations, arc length and speed

1. For the following parametric curves, find an equation for the tangent to the curve at the specified value of the parameter.

   (a) $x = e^{\sqrt{t}}$, $y = t - \ln(t^2)$ at $t = 1$.

   (b) $x = \cos(\theta) + \sin(2\theta)$, $y = \sin(\theta)$ at $\theta = \pi/2$.

2. For the following parametric curve, find dy/dx.

   (a) $x = e^{\sqrt{t}}$, $y = t + e^{-t}$.

   (b) $x = t^3 - 12t$, $y = t^2 - 1$.

   (c) $x = 4\cos(t)$, $y = \sin(2t)$.

3. Find the arc length of the following curves.

   (a) $x = 1 + 3t^2$, $y = 4 + 2t^3$, $0 \le t \le 1$.

   (b) $x = 4\cos(t)$, $y = 4\sin(t)$, $0 \le t \le 2\pi$.

   (c) $x = 3t^2$, $y = 4t^3$, $1 \le t \le 3$.

4. What is the speed of the parametrization $c(t) = (x(t), y(t))$? Use this to find the minimum speed of a particle with trajectory $c(t) = (t^2, 2\ln(t))$, for $t > 0$.

5. Suppose you wrap a string around a circle. If you unwind the string from the circle while holding it taut, the end of the string traces out a curve called the *involute* of the circle. Suppose you have a circle of radius r centered at the origin, with the end of the string all the way wrapped up resting at the point $(r, 0)$. As you unwrap the string, define θ to be the angle formed by the x-axis and the line segment from the center of the circle to the point up to which you have unwrapped the string.

   (a) Draw a picture and label θ.

   (b) Show that the parametric equations of the involute are given by $x = r(\cos\theta + \theta\sin\theta)$, $y = r(\sin\theta - \theta\cos\theta)$.

   (c) Find the length of the involute for $0 \le \theta \le 2\pi$.

6. Consider the line through $P = (1, 0)$ and $Q = (7, 8)$. Find parametrizations of this line with the following speeds.

   (a) $s'(t) = 1$

   (b) $s'(t) = 3$

   (c) $s'(t) = t$

   (d) $s'(t) = t^2$