Worksheet # 10: Product and Quotient Rules

1. Show by way of example that, in general,

$$\frac{d(f \cdot g)}{dx} \neq \frac{df}{dx} \cdot \frac{dg}{dx}$$

and

$$\frac{d\left(\dfrac{f}{g}\right)}{dx} \neq \frac{\dfrac{df}{dx}}{\dfrac{dg}{dx}}.$$

2. State the quotient and product rule and be sure to include all necessary hypotheses.

3. Compute the first derivative of each of the following:

   (a) $f(x) = \dfrac{\sqrt{x}}{x-1}$

   (b) $f(x) = (3x^2 + x)e^x$

   (c) $f(x) = \dfrac{e^x}{2x^3}$

   (d) $f(x) = (x^3 + 2x + e^x)\left(\dfrac{x-1}{\sqrt{x}}\right)$

   (e) $f(x) = \dfrac{2x}{4+x^2}$

   (f) $f(x) = \dfrac{ax+b}{cx+d}$

   (g) $f(x) = \dfrac{(x^2+1)(x^3+2)}{x^5}$

4. Calculate the first three derivatives of $f(x) = xe^x$ and use these to guess a general formula for $f^{(n)}(x)$, the n-th derivative of f.

5. Find an equation of the tangent line to the given curve at the specified point.

   (a) $y = x^2 + \dfrac{e^x}{x^2+1}$ at the point $x = 3$

   (b) $y = 2xe^x$, $x = 0$

6. Suppose that $f(2) = 3$, $g(2) = 2$, $f'(2) = -2$, and $g'(2) = 4$. For the following functions, find $h'(2)$.

   (a) $h(x) = 5f(x) + 2g(x)$

   (b) $h(x) = f(x)g(x)$

   (c) $h(x) = \dfrac{f(x)}{g(x)}$

   (d) $h(x) = \dfrac{g(x)}{1+f(x)}$