Worksheet # 12: Chain Rule

1. (MA 113 Exam II, problem 9, Spring 2009).
   (a) Carefully state the chain rule. Use complete sentences.
   (b) Suppose f and g are differentiable functions so that $f(2) = 3$, $f'(2) = -1$, $g(2) = 1/4$, and $g'(2) = 2$. Find each of the following:
      i. $h'(2)$ where $h(x) = \sqrt{[f(x)]^2 + 7}$.
      ii. $l'(2)$ where $l(x) = f(x^3 \cdot g(x))$.
2. Differentiate each of the following and simplify your answer.
   (a) $f(x) = \sqrt[3]{2x^3 + 7x + 3}$
   (b) $g(t) = \tan(\sin t)$
   (c) $h(u) = \sec^2 u + \tan^2 u$
   (d) $f(x) = e^{(3x^2+x)}$
   (e) $g(x) = \sin(\sin(\sin x))$
3. Find an equation of the tangent line to the curve at the given point.
   (a) $f(x) = x^2 e^{3x}$, $x = 2$
   (b) $f(x) = \sin x + \sin^2 x$, $x = 0$
4. If $h(x) = \sqrt{4 + 3f(x)}$ where $f(1) = 7$ and $f'(1) = 4$, find $h'(1)$.
5. Let $h(x) = f \circ g(x)$ and $k(x) = g \circ f(x)$ where some values of f and g are given by the table

x	f(x)	g(x)	f'(x)	g'(x)
-1	4	4	-1	-1
2	3	4	3	-1
3	-1	-1	3	-1
4	3	2	2	-1

   Find: $h'(-1), h'(3)$ and $k'(2)$.
6. Find all x values so that $f(x) = 2\sin x + \sin^2 x$ has a horizontal tangent at x.
7. Comprehension check for derivatives of trigonometric functions.
   (a) True or false? If $f'(x) = -\sin(\theta)$ then $f(\theta) = \cos(\theta)$.
   (b) True or False? If θ is one of the non-right angles in a right triangle and $\sin(\theta) = \frac{2}{3}$ then the hypotenuse of the triangle must have length 3.
   (c) Let $f(\theta) = \sin(\theta)$. Find $f^{(435)}(\theta)$.
   (d) Differentiate both sides of the identity

$$\tan x = \frac{\sin x}{\cos x}$$

   to obtain a new trigonometric identity.