Worksheet # 14: Growth, Decay, and Rates of Change

1. Comprehension check for exponential functions.

   (a) Explain the intuition behind the simple population growth model
   $$\frac{dP}{dt} = kP$$
   where k is a positive constant. Describe some situations where this model may break down.

   (b) What is the unique (only) function satisfying $f'(x) = f(x)$ and $f(0) = 1$?

   (c) Find a function $f(x)$ such that $f'(x) = 3f(x)$ and $f(0) = 15$.

2. An explorer brought two rabbits (male and female) to a small island. Based on 30 years of data, the Rabbit Research Group has concluded that the rabbit population on the island doubles every year. Set up the proportional growth rate population equation and use it to predict the number of rabbits for 10, 50, and 100 years. What might be wrong with using this model to predict population values for large values of t?

3. A bacteria culture initially contains 100 cells and grows at a rate proportional to its size. After an hour the population has increased to 420.

   (a) Find an expression for the number of bacteria after t hours.

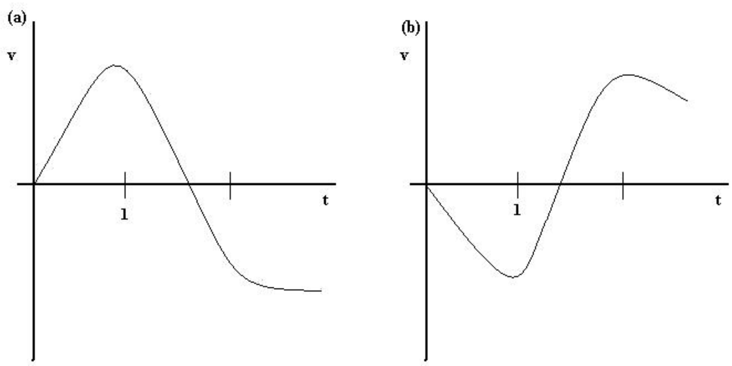
   (b) Find the number of bacteria after 3 hours.

   (c) Find the rate of growth after 3 hours.

   (d) When will the population reach 10,000 bacteria?

4. A sample of a chemical compound decayed to 95.47 % of its original mass after one year. What is the half life of the compound? How long would it take for the compound to decay to 20 % of its original mass.

5. Graphs of the **velocity** functions of two particles are shown, where t is measured in seconds. When is each particle speeding up? When is it slowing down? Explain.



6. A man uses a helium tank to inflate a large balloon. The balloon's surface area is given by $S = 4\pi r^2$ and its volume is given by $V = \frac{4}{3}\pi r^3$.

   (a) Find the rate of increase in surface area with respect to the radius when the diameter of the balloon is 2 ft.

   (b) Suppose the radius of the balloon at time t seconds is $r(t) = 2t + 1$. Find the rate of increase in surface area with respect to time when $t = 1$ sec.

   (c) Show that if the volume of the balloon is decreasing at a rate (with respect to time) proportional to its surface area, then the radius of the balloon is shrinking at a constant rate.