Worksheet # 16: Review for Exam II

1. State:

   (a) The product rule and quotient rule.

   (b) The chain rule.

2. Compute the first derivative of each of the following functions.

   (a) $f(x) = \cos(4\pi x^3) + \sin(3x + 2)$

   (b) $b(x) = x^4 \cos(3x^2)$

   (c) $y(x) = e^{\sec 2\theta}$

   (d) $k(x) = \ln(7x^2 + \sin(x) + 1)$

   (e) $u(x) = (\arcsin 2x)^2$

   (f) $h(x) = \dfrac{8x^2 - 7x + 3}{\cos(2x)}$

   (g) $m(x) = \sqrt{x} + \dfrac{1}{\sqrt[3]{x^4}}$

   (h) $q(x) = \dfrac{e^x}{1 + x^2}$

   (i) $n(x) = \cos(\tan x)$

   (j) $w(x) = \arcsin x \cdot \arccos x$

3. The tangent line to $f(x)$ at $x = 3$ is given by $y = 2x - 4$. Find the tangent line to $g(x) = \dfrac{x}{f(x)}$ at $x = 3$. Put your answer in slope-intercept form.

4. (MA 113 Exam II, Problem 6, Fall 2008). Consider the curve $xy^3 + 12x^2 + y^2 = 24$. Assume this equation can be used to define y as a function of x (i.e. $y = y(x)$) near $(1, 2)$ with $y(1) = 2$. Find the equation of the tangent line to this curve at $(1, 2)$.

5. Let x be the angle in the interval $(-\pi/2, \pi/2)$ so that $\sin x = \dfrac{-3}{5}$. Find: $\sin(-x), \cos(x)$, and $\cot(x)$.

6. (Adapted from MA 113 Exam II, Problem 7, Fall 2008). The growth rate of the population in a bacteria colony at time t obeys the *differential equation*

   $$P'(t) = kP(t)$$

   where k is a constant and t is measured in years.

   (a) Let A be a constant. Show that the function $P(t) = Ae^{kt}$ satisfies the differential equation.

   (b) If the colony initially has 100 bacteria and two years later has 200 bacteria, determine the values of A and k.

   (c) Suppose $P(t) = 100e^{.001t}$. When will the colony have 100,000 bacteria?

7. (MA 113 Exam II, Problem 9, Spring 2009). Suppose f and g are differentiable functions such that $f(2) = 3$, $f'(2) = -1$, $g(2) = 1/4$, and $g'(2) = 2$. Find:

   (a) $h'(2)$ where $h(x) = \ln([f(x)]^2)$;

   (b) $l'(2)$ where $l(x) = f(x^3 \cdot g(x))$.

8. (MA 113 Exam II, Problem 9, Spring 2007). Abby is north of Oakville and driving north along Road A. Boris is east of Oakville and driving west on Road B. At 11:57 AM, Boris is 5 km east of Oakville and traveling west at a speed of 60 kmph and Abby is 10 km north of Oakville and traveling north at a speed of 50 kmph.

(a) Make a sketch showing the location and direction of travel for Abby and Boris.

(b) Find the rate of change of the distance between Abby and Boris at 11:57 AM.

(c) At 11:57 AM, is the distance between Abby and Boris increasing, decreasing, or not changing?

9. (MA 113 Exam II, Problem 10, Fall 2008). The function $\arctan x$ is defined by $y = \arctan x$, if and only if $x = \tan y$, $-\pi/2 < y < \pi/2$. Use implicit differentiation to find the derivative of $\arctan x$. [Hint: use a trigonometric identity.]