Worksheet # 17: Maxima and Minima

1. Define the following terms:
   - Critical number.
   - Local maximum.
   - Absolute maximum.
2. Sketch:
   (a) The graph of a function defined on $(-\infty, \infty)$ with three local maxima, two local minima, and no absolute minima.
   (b) The graph of a continuous function with a local maximum at $x = 1$ but which is not differentiable at $x = 1$.
   (c) The graph of a function on $[-1, 1)$ which has a local maximum but not an absolute maximum.
   (d) The graph of a function on $[-1, 1]$ which has a local maximum but not an absolute maximum.
   (e) The graph of a discontinuous function defined on $[-1, 1]$ which has both an absolute minimum and absolute maximum.
3. Find the critical numbers for the following functions
   (a) $f(x) = x^3 + x^2 + 1$
   (b) $f(x) = \dfrac{x}{x^2 + 3}$
   (c) $f(x) = |5x - 1|$
4. Given a continuous function on a closed interval $[a, b]$, carefully describe the method you would use to find the absolute minimum and maximum value of the function.
5. Use the extreme value theorem to find the absolute maximum and absolute minimum value of the following function on the given intervals. Specify the values where these extrema occur.
   (a) (MA 113 Exam III, Problem 2, Fall 2008). $f(x) = 2x^3 - 3x^2 - 12x + 1$, $[-2, 3]$
   (b) (MA 113 Exam III, Problem 2, Spring 2009). $f(t) = t + \sqrt{1 - t^2}$, $[-1, 1]$.
6. Comprehension check.
   (a) True or false? An absolute maximum is always a local maximum.
   (b) True or false? If $f'(c) = 0$ then f has a local maximum or local minimum at c.
   (c) True or false? If f is differentiable and has a local maximum or minimum at $x = c$ then $f'(c) = 0$.
   (d) A function continuous on an open interval may not have an absolute minimum or absolute maximum on that interval. Give an example of continuous function on $(0, 1)$ which has no absolute maximum.