10-28-2009

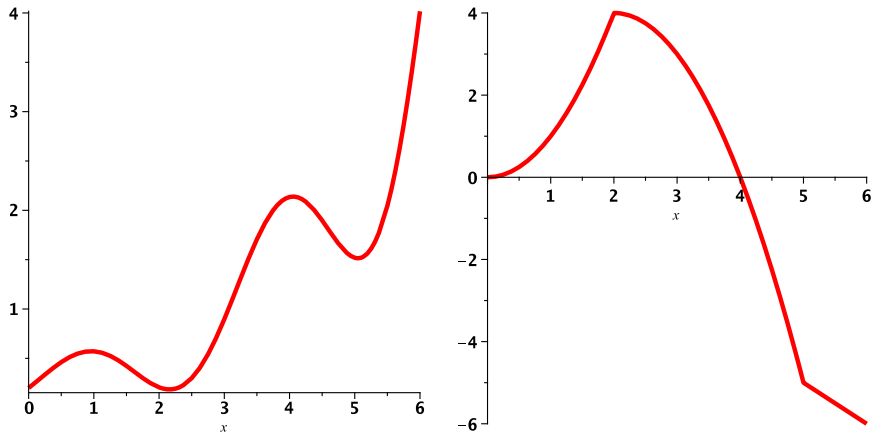
Worksheet # 19: Asymptotes and Curve Sketching

1. (a) Define the terms **horizontal asymptote** and **vertical asymptote**.
   (b) Explain the difference between $\lim_{x\to -3} f(x) = \infty$ and $\lim_{x\to\infty} f(x) = -3$.
   (c) Explain what $\lim_{x\to\infty} f(x) = 150$ means.
   (d) Explain what $\lim_{x\to 150} f(x) = 150$ means.
   (e) Explain how to use the first derivative test to identify and classify local extrema of the differentiable function $f(x)$.
   (f) Explain how to use the second derivative test to identify and classify local extrema of the twice differentiable function $f''(x)$. Does the test always work? What should you do if it fails?

2. (MA 113 Exam III, Problem 1, Spring 2009). Consider the function $f(x) = 2x^3 - 3x^2 - 36x + 4$ on $(-\infty, \infty)$.
   (a) Find the critical point(s) of f.
   (b) Find the intervals of increase and decrease for f.
   (c) Find the local extrema of f.

3. (MA 113 Exam III, Problem 3, Spring 2009). Consider the function $f(x) = 2x + \sin x$ on $(-\pi, 2\pi)$.
   (a) Find the interval(s) of concavity of the graph of $f(x)$; show your work.
   (b) Find the point(s) of inflection of the graph of $f(x)$; justify your work.

4. For each graph of the function f:



   (a) Find the open interval(s) where f is increasing.
   (b) Find the open interval(s) where f is decreasing.
   (c) Find the open interval(s) where f is concave up.
   (d) Find the open interval(s) where f is concave down.
   (e) Identify all points of inflection.
   (f) Identify and classify all local extrema on $[0, 6]$.

5. Find the local maximum and minimum values of $f(x) = \dfrac{x}{x^2 + 4}$ using the first derivative test.

6. Find the local maximum and minimum values of $f(x) = x^5 - 5x + 4$ using the second derivative test.

7. Sketch the graph of a function f with all of the following properties.
   - $\lim_{t\to\infty} f(t) = 2$
   - $\lim_{t\to -\infty} f(t) = 0$

- $\lim_{t \to 0^+} f(t) = \infty$
- $\lim_{t \to 0^-} f(t) = -\infty$
- $\lim_{t \to 4} f(t) = 3$
- $f(4) = 6$

8. Evaluate the following limits, if they exist. If a limit does not exist, explain why.

   (a) $\lim_{t \to \infty} \dfrac{3t^2 - 7t}{t - 8}$

   (b) $\lim_{t \to \infty} \dfrac{2t^2 - 6}{t^4 - 8t + 9}$

   (c) $\lim_{t \to -\infty} \dfrac{t}{t^6 - 4t^2}$

   (d) $\lim_{t \to -\infty} 3$

   (e) $\lim_{t \to \pm\infty} \dfrac{5t^3 - 7t^2 + 9}{t^2 - 8t^3 - 8999}$

   (f) $\lim_{u \to \infty} \sqrt{16u^2 - u} - 4u$