Worksheet # 2: Functions and Inverse Functions; Logarithms

1. (MA 113 Exam I, Problem 2, Spring 2009). Consider the function $f(x) = \dfrac{4x+1}{3x-2}$. Determine the inverse function of f.

2. Let $f(x) = x^3 + 1$ and $g(x) = \sqrt{x}$. Find $(f \circ g)(x)$ and $(g \circ f)(x)$ and specify their domains.

3. Suppose the graph of $f(x)$ is given. Write an equation for the graph obtained by first shifting the graph of $f(x)$ up 3 units and left by 2 units, and then compressing the resulting graph horizontally by a factor of 10.

4. Suppose the graph of $g(x)$ is given by the equation $g(x) = f(2x - 5) + 7$. In terms of standard transformations describe how to obtain $g(x)$ from the graph of $f(x)$.

5. Find the domain and range of the following functions.

   (a) $f(x) = 15$
   (b) $f(x) = \sqrt{x^2 + 2x + 1}$
   (c) $f(x) = \sqrt{x^2 - 2x - 3}$
   (d) $f(x) = \dfrac{x}{|x|}$

6. Profit is the difference between total revenues and total costs. Suppose that Company W produces good Y. Let x denote the quantity of good Y sold. Suppose

$$R(x) = 15x$$

   and

$$C(x) = \frac{1}{10}x^2 + x + 30$$

   are the company's revenue and cost functions respectively for sales of this good.

   (a) Find the company's profit function $P(x)$.
   (b) Company W would really like to know how much of good Y they must sell to break even. Find the quantity x of good Y that the company must sell to make neither a profit nor a loss.

7. Compute each of the following logarithms exactly. Do not use a calculator.

   (a) $\log_{10} \sqrt{10^3}$
   (b) $\log_3(1/27)$
   (c) $\log_2 6 - \log_2 15 + \log_2 20$
   (d) $\log_{10}(\log_{10}(\log_{10}(10^{10^{100}})))$

8. Express each of the following as a single logarithm.

   (a) $\log_{10}(5) - \log_{10}(3) + \log_{10}(2)$
   (b) $\log_3(a + b) - 15\log_3(c) + 17\log_3(d)$

9. Solve the following equations for x

   (a) $10^{2x+1} - 7 = 0$
   (b) $\log_2(x) + \log_2(x - 1) = 1$
   (c) $3^{ax} = C \cdot 3^{bx}$, $a \neq b$.

10. True or false?

    (a) Every function has an inverse.
    (b) Every function will pass the vertical line test.
    (c) Every function will pass the horizontal line test.
    (d) $f \circ g(x) = g \circ f(x)$.
    (e) There is a function whose graph is an oval.
    (f) No function can be both even and odd.