Worksheet # 20: L'Hospital's Rule and Curve Sketching

1. Carefully, state L'Hospital's Rule.

2. Compute the following limits. Use l'Hospital's Rule where appropriate but first check that no easier method will solve the problem.

   (a) $\displaystyle\lim_{x\to 1} \frac{x^9 - 1}{x^5 - 1}$

   (b) $\displaystyle\lim_{x\to 0} \frac{\sin 4x}{\tan 5x}$

   (c) $\displaystyle\lim_{x\to 2} \frac{x^2 + x - 6}{x - 2}$

   (d) $\displaystyle\lim_{x\to \infty} \frac{e^x}{x^3}$

   (e) $\displaystyle\lim_{x\to -\infty} x^2 e^x$

   (f) $\displaystyle\lim_{x\to \infty} x^3 e^{-x^2}$

3. Choose a and b so that

$$\lim_{x\to 0} \frac{\sin 3x + ax + bx^3}{x^3} = 0.$$

4. (MA 113 Exam III, Problem 11, Spring 2008). Sketch the graph of a function $f(x)$ defined for $x > 0$ such that

   (a) $\displaystyle\lim_{x\to 0^+} f(x) = 3$,

   (b) $f(2) = f(4) = 2, f(3) = 4$,

   (c) $\displaystyle\lim_{x\to \infty} f(x) = f(1) = 1$,

   (d) $f''(x)$ exists and is continuous for all $x > 0$,

   (e) $f'(1) = f'(3) = f''(2) = f''(4) = 0$, and $f'(x)$ and $f''(x)$ are not zero for all other values of x.

5. Sketch the graph of a function which satisfies all of the following properties.

   - $f(1) = f'(1) = 0$
   - $\displaystyle\lim_{x\to 2^+} f(x) = \infty$
   - $\displaystyle\lim_{x\to 2^-} f(x) = -\infty$
   - $\displaystyle\lim_{x\to 0} f(x) = -\infty$
   - $\displaystyle\lim_{x\to -\infty} f(x) = \infty$
   - $\displaystyle\lim_{x\to \infty} f(x) = 0$
   - $f''(x) > 0$ when $x > 2$
   - $f''(x) < 0$ when $x < 0$ and $0 < x < 2$.

6. (a) Outline a procedure for sketching the curve $y = f(x)$ using the tools of calculus.

   (b) Sketch the following curves using the procedure you described above. Check your answers with a calculator.

      i. $y = 8x^2 - x^4$

      ii. $y = \dfrac{x}{x^2 - 1}$