Worksheet # 24: Review for Exam III

1. Provide a full statement of the following theorems and definitions.
   (a) The Mean Value Theorem (MVT)
   (b) Local max/min
   (c) Absolute max/min
   (d) L'Hospital's Rule
   (e) Antiderivative

2. (a) Describe in words and diagrams how to use the first derivative test to identify and classify extrema of a function $f(x)$.
   (b) Use the first derivative test to classify the extrema of the function $f(x) = 2x^3 + 3x^2 - 72x - 47$.

3. (a) Describe how to use the second derivative test. When does the test fail and what can you do if this happens?
   (b) Use the second derivative test to classify the extrema of $4x^3 + 3x^2 - 6x + 1$.

4. (a) Explain how to use the extreme value theorem (p. 272) to find the absolute maximum and absolute minimum of a continuous function $f(x)$ on a closed interval $[a, b]$.
   (b) (MA 113 Exam III, Problem 2, Spring 2009). Find the absolute minimum of the function

$$f(t) = t + \sqrt{1 - t^2}$$

   on the interval $[-1, 1]$. Be sure to specify the value of t where the minimum is attained.

5. Evaluate the following limits.
   (a) $\displaystyle\lim_{x\to-\infty} \frac{x+2}{\sqrt{9x^2+1}}$
   (b) $\displaystyle\lim_{x\to 0^+} x^2 \ln x$
   (c) $\displaystyle\lim_{x\to\infty} x^2 e^x$

6. Find the most general antiderivative for each of the following.
   (a) $f(x) = 5x^{10} + 7x^2 + x + 1$
   (b) $g(x) = 2\cos(2x + 1)$
   (c) $h(x) = \dfrac{1}{2x+1}$, where $2x + 1 > 0$

7. (MA 113 Exam III, Problem 10, Spring 2001). Suppose a rectangle has one side on the x-axis and its other two vertices above the x-axis on the curve $y = 80 - x^4$. Find the dimensions of the rectangle satisfying these conditions and of largest possible area. Be sure to explain how you know you have found the absolute extreme value.

8. If $f(2) = 30$ and $f'(x) \geq 4$ for $2 \leq x \leq 6$, how small can $f(6)$ be?

9. Let $f(x) = (x - 1) + \dfrac{1}{x-1}$.
   (a) Find the y-intercept(s) of the graph of f.
   (b) Find all vertical asymptotes to the graph of f.
   (c) Compute $f'(x)$ and give the domain of $f'(x)$.
   (d) Use the first derivative to determine the intervals of increase and decrease for f and find all local maxima and local minima for f.
   (e) Compute $f''(x)$ and give the domain of $f''(x)$.

(f) Use the second derivative to find intervals of concavity for f.

(g) Sketch the graph of f and label all local extrema. Sketch the vertical asymptote(s) with dashed lines.

10. Identify each of the following as true or false.

(a) A point in the domain of f where $f'(x)$ does not exist is a critical point.

(b) Every continuous function on a closed interval will have an absolute minimum and an absolute maximum.

(c) If $f'(c) = 0$, f will have either a maximum or a minimum at c.

(d) An inflection point is an ordered pair.

(e) If $f'(c) = 0$ and $f''(c) > 0$ then c is a local minimum.

(f) If $f''(c) = 0$ in the second derivative test, we must use some other method to determine if c is a min or max.

(g) A continuous function on $[a, b]$ will always have a local maximum or minimum at its endpoints.