Worksheet # 27: The Fundamental Theorem of Calculus

1. (MA 113 Exam IV, Problem 9, Spring 2008).

   (a) State both parts of the fundamental theorem of calculus. Use complete sentences.

   (b) Consider the function f on $[1, \infty)$ defined by $f(x) = \int_1^x \sqrt{t^5 - 1}dt$. Argue that f is increasing.

   (c) Find the derivative of the function $g(x) = \int_1^{x^3} \sqrt{t^5 - 1}dt$ on $(1, \infty)$.

2. Use Part I of the fundamental theorem of calculus to find the derivative of the following functions.

   (a) $g(x) = \displaystyle\int_1^x \left(2 + t^4\right)^5 \, dt$

   (b) $F(x) = \displaystyle\int_x^4 \cos\left(t^5\right) \, dt$

   (c) $h(x) = \displaystyle\int_0^{x^2} \sqrt[3]{1 + r^3} \, dr$

   (d) $y(x) = \displaystyle\int_{1/x^2}^0 \sin^3 t \, dt$

   (e) $G(x) = \displaystyle\int_{\sqrt{x}}^{x^2} \sqrt{t} \sin t \, dt$

3. Use Part II of the fundamental theorem of calculus to evaluate the following integrals or explain why the theorem does not apply.

   (a) $\displaystyle\int_{-2}^5 6x \, dx$

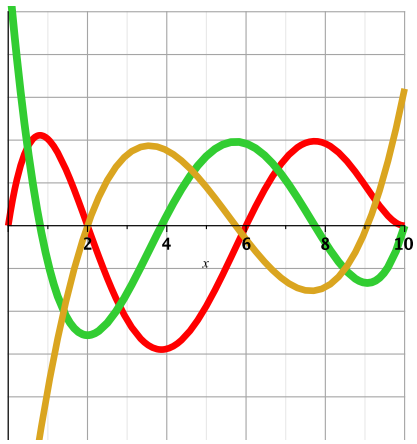
   (b) $\displaystyle\int_{-2}^7 \frac{1}{x^5} \, dx$

   (c) $\displaystyle\int_{-1}^1 e^{u+1} du$

   (d) $\displaystyle\int_0^{\pi/4} \sec^2 t \, dt$

   (e) $\displaystyle\int_{\pi/3}^{\pi/6} \frac{\sin 2x}{\sin x} \, dx$

4. Below is pictured the graph of the function $f(x)$, its derivative $f'(x)$, and an antiderivative $\int f(x)\,dx$. Identify f, f' and $\int f\,dx$.



5. Evaluate the following limits by first recognizing the sum as a Riemann sum.

(a) $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{i^3}{n^4}$

(b) $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{\sqrt{3+\frac{i}{n}}}{n}$

(c) $\lim_{n\to\infty} \sum_{i=1}^{n} 2\frac{(2+\frac{2i}{n})^2}{n}$