Worksheet # 3: Tangents and Velocity

1. Sketch the graphs of the following functions using your knowledge of basic functions and transformations. Then sketch the tangent line to the curve at the specified point.

   (a) $f(x) = x^2 + 1$, $x = 2$

   (b) $f(x) = -|x| + 3$, $x = -1$

   (c) $f(x) = (x-2)^3 - 1$, $x = 2$

   (d) $f(x) = 2^{x-1} + 1$, $x = 1$.

2. (Adapted from MA 113 Exam I, Problem 6, Spring 2009). A particle is moving along a straight line so that its position at time t seconds is given by $s(t) = 4t^2 - t$.

   (a) Find the average velocity of the particle over the time interval $[1, 2]$.

   (b) Determine the average velocity of the particle over the time interval $[2, t]$ where $t > 2$. Simplify your answer. [Hint: Factor the numerator.]

   (c) Based on your answer in (b) can you guess a value for the instantaneous velocity of the particle at $t = 2$?

3. Let $x(t)$ be the function which describes the position of a particle traveling along the x-axis. Suppose the point $(15, 6)$ is on the graph of $x(t)$ and the tangent line at this point is given by $y = -3$. At time $t = 15$, determine the particle's position and instantaneous velocity.

4. (Problem 4, p. 87 in the text.) The point $P(3, 1)$ lies on the curve $y = \sqrt{x-2}$.

   (a) If Q is the point $(x, \sqrt{x-2})$, find a formula for the slope of the secant line PQ.

   (b) Using your formula from part (a) and a calculator, find the slope of the secant line PQ for the following values of x. [1] Keep 4 decimal places of accuracy and be careful with rounding.

      i. 2.9
      ii. 2.99
      iii. 2.999
      iv. 3.1
      v. 3.01
      vi. 3.001

   (c) Using the results of part (b), guess the value of the slope of the tangent line to the curve at $P(3, 1)$.

   (d) Using the slope from part (c), find the equation of the tangent line to the curve at $P(3, 1)$.

5. (Adapted from problem 5, p. 87 in the text.) If a ball is thrown in the air with a velocity of 40 ft/s, its height in feet t seconds later is given by $f(t) = 40t - 16t^2$.

   (a) Using a calculator, find the average velocity of the ball for the time period beginning when $t = 2$ and lasting

      i. 0.5 second
      ii. 0.1 second
      iii. 0.05 second
      iv. 0.01 second

   (b) Estimate the instantaneous velocity when $t = 2$.

   (c) Find a general formula for the average velocity of the ball for the time period beginning at t and lasting h seconds. Simplify your answer.

[1] TI-8X calculator tip: Hit the "y=" button and put your formula from part a.) in, say, the y_1 position. Then go to the home screen, access the y-vars menu, and use it to type $y_1(x)$ to find the value of y_1 at the point x. You could also use the table feature.

(d) Based on your answer in (c), can you guess a general formula for the instantaneous velocity at time t? [Hint: What does the result in (c) look like as h gets very close to 0?]

6. Let $s(t)$ describe the position of a particle traveling along the x-axis at time t. Let $v(t)$ be the particle's instantaneous velocity and $a(t)$ be the instantaneous acceleration function at time t. Determine if the following statements are true or false.

   (a) If $v(t) = 0$ then the particle is at rest at time t.

   (b) If $s(t) = 0$ then the particle is at the origin at time t.

   (c) If $a(t) > 0$ then the particle must be speeding up at time t.

   (d) If $a(t) = 0$ and $s(t) = 0$, the particle will remain at the origin.

   (e) If $a(t) > 0$ and $v(t) = 0$ at time t, the particle will soon begin traveling to the right.

   (f) If $v(t)$ is constant for all t, then $a(t) = 0$.

   (g) Suppose $v(t) > 0$ and $s(t) > 0$ for all time values. Then the particle will stay to the right of the origin forever.