

## Worksheet # 4: Introduction to Limits

1. Comprehension check.

(a) In words, describe what

$$\lim_{x \rightarrow a} f(x) = L$$

means.

(b) How can one-sided limits help you to determine if a limit exists?

(c) In words, what does

$$\lim_{x \rightarrow a} f(x) = \infty$$

mean?

(d) Suppose  $\lim_{x \rightarrow 1} f(x) = 2$ . Does  $f(1) = 2$ ?

(e) Suppose  $f(1) = 2$ . What can be said about  $\lim_{x \rightarrow 1} f(x)$ ?

2. Let

$$f(x) = \begin{cases} x^2, & x \leq 0 \\ x - 1, & 0 < x, x \neq 2 \\ -3, & x = 2 \end{cases} .$$

(a) Sketch the graph of  $f$ .

(b) Compute the following.

i.  $\lim_{x \rightarrow 0^-} f(x)$

ii.  $\lim_{x \rightarrow 0^+} f(x)$

iii.  $\lim_{x \rightarrow 0} f(x)$

iv.  $f(0)$

v.  $\lim_{x \rightarrow 2^-} f(x)$

vi.  $\lim_{x \rightarrow 2^+} f(x)$

vii.  $\lim_{x \rightarrow 2} f(x)$

viii.  $f(2)$

3. In the following, sketch the functions and use the sketch to compute the limit.

(a)  $\lim_{x \rightarrow 3} \pi$

(b)  $\lim_{x \rightarrow \pi} x$

(c)  $\lim_{x \rightarrow a} |x|$

(d)  $\lim_{x \rightarrow 3} 2^x$

4. Compute the following limits or explain why they fail to exist:

(a)  $\lim_{x \rightarrow -3^+} \frac{x+2}{x+3}$

(b)  $\lim_{x \rightarrow -3^-} \frac{x+2}{x+3}$

(c)  $\lim_{x \rightarrow -3} \frac{x+2}{x+3}$

(d)  $\lim_{x \rightarrow 0^-} \frac{1}{x^3}$

5. (Problem 40, p. 99 in the text). In the theory of relativity, the mass of a particle with velocity  $v$  is:

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

where  $m_0$  is the mass of the particle at rest and  $c$  is the speed of light. What happens as  $v \rightarrow c^-$ ?

6. Let

$$f(x) = \begin{cases} 2x + 2, & x > -2 \\ a, & x = -2 \\ kx, & x < -2 \end{cases} .$$

Find  $k$  and  $a$  so that  $\lim_{x \rightarrow -2} f(x) = f(-2)$ .