Assignment 3 for MA 113 - Calculus I (Spring 2010)

February 11, 2010

Instructions: The purpose of this and subsequent assignments is to develop your ability to formulate and communicate a mathematical argument showing step-by-step reasoning.

Please give a complete, well-written solution to each of the following problems. Your work will be graded for accuracy, completeness, and grammatically correct English.

Your solutions should be neat and legible, stapled, and your name should appear on each sheet. Moreover, on page 1 of your solution, please also indicate your *section number* to ensure that you will receive proper credit for the assignment.

Due Date: Your completed solutions are due on **Friday, February 19, 2010**, at the beginning of lecture.

(1) (3 Points) Consider the function $f(x) = 2e^x + ax^4 + 5$, where a is a constant. Determine a such that the tangent line to $y = f(x)$ at the point $(1, f(1))$ has slope 2.

For the following exercise, it might be helpful to recall the quadratic formula. The roots of the quadratic equation $ax^2 + bx + c = 0$ are

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

The quantity inside the radical, $b^2 - 4ac$, is called the discriminant. It follows that we have two real roots if the discriminant is positive, one real root if the discriminant is 0, and no real roots if the discriminant is negative.

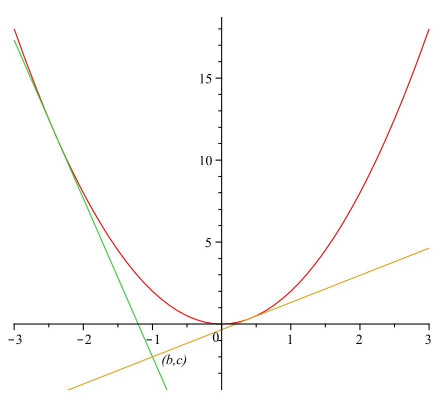
(2) (7 Points) Consider the parabola $y = 2x^2$, and let (b, c) be a point which may or may not lie on the parabola.

(a) Write the equation of the tangent line to the parabola $y = 2x^2$ at $(a, 2a^2)$.

(b) Make a conjecture as to how many tangent lines of the parabola pass through the given point (b, c). How does the answer depend on the point (b, c)? See the picture below for one option.

(c) Give conditions on b and c which tell us whether we have exactly 0, 1 or 2 tangent lines through (b, c).
[Hint: If we require the tangent line in part (a) to pass through the point (b, c), we obtain an equation for a. Write out this equation and solve for a. This will give you the desired conditions on b and c.]

(d) Interpret your answers in (c) geometrically. What do these conditions tell us about the location of the point (b, c) with respect to the parabola?



Bonus Problem: (2 Points) How can you plant 12 trees in 6 straight lines such that each line contains exactly 4 trees?