

## SOME FACTS EVERY STUDENT OF CALCULUS SHOULD KNOW.

Below are some basic facts from trigonometry that all of us should know. Note that in addition to memorizing certain formulae, you should also understand how to derive consequences of these formulae.

1. The definitions of the sine and cosine functions using the unit circle.
2. The identities  $\sin(-x) = -\sin x$  and  $\cos x = \cos(-x)$  which tell us that sin is an odd function and cos is an even function.
3. The definitions of tan, cot, sec and csc in terms of sin and cos.
4. The trigonometric functions for the special angles  $0, \pi/6, \pi/4, \pi/3$ , and any angle obtained by adding a multiple of  $\pi/2$ .
5. The Pythagorean identity  $\sin^2 x + \cos^2 x = 1$  and how to derive the identities  $1 + \tan^2 x = \sec^2 x$  and  $1 + \cot^2 x = \csc^2 x$ .
6. The addition formulae for sin and cos:

$$\sin(x + y) = \sin x \cos y + \cos x \sin y$$

and

$$\cos(x + y) = \cos x \cos y - \sin x \sin y.$$

7. Consequences of the addition formulae such as the subtraction and the double-angle formulae for sin and cos and the co-function identities:  $f(\pi/2 - x) = \operatorname{cof}f(x)$  where  $f$  may be sin, tan and sec.

Values of the trigonometric functions at special angles.

$\theta$	$\cos \theta$	$\sin \theta$	$\tan \theta$	$\cot \theta$	$\sec \theta$	$\csc \theta$
0	1	0	0	undefined	1	undefined
$\frac{\pi}{6}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{3}$	$\sqrt{3}$	$\frac{2\sqrt{3}}{3}$	2
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	1	$\sqrt{2}$	$\sqrt{2}$
$\frac{\pi}{3}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\sqrt{3}$	$\frac{\sqrt{3}}{3}$	2	$\frac{2\sqrt{3}}{3}$
$\frac{\pi}{2}$	0	1	undefined	0	undefined	1
$\frac{2\pi}{3}$	$-\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$-\sqrt{3}$	$-\frac{\sqrt{3}}{3}$	-2	$\frac{2\sqrt{3}}{3}$
$\frac{3\pi}{4}$	$-\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	-1	-1	$-\sqrt{2}$	$\sqrt{2}$
$\frac{5\pi}{6}$	$-\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$-\frac{\sqrt{3}}{3}$	$-\sqrt{3}$	$-\frac{2\sqrt{3}}{3}$	2
$\pi$	-1	0	0	undefined	-1	undefined
$\frac{7\pi}{6}$	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$\frac{\sqrt{3}}{3}$	$\sqrt{3}$	-2	$-\frac{2\sqrt{3}}{3}$
$\frac{5\pi}{4}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$	1	1	$-\sqrt{2}$	$-\sqrt{2}$
$\frac{4\pi}{3}$	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$\sqrt{3}$	$\frac{\sqrt{3}}{3}$	-2	$-\frac{2\sqrt{3}}{3}$
$\frac{3\pi}{2}$	0	-1	undefined	0	undefined	-1
$\frac{5\pi}{3}$	$\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$-\sqrt{3}$	$-\frac{\sqrt{3}}{3}$	2	$-\frac{2\sqrt{3}}{3}$
$\frac{7\pi}{4}$	$\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$	-1	-1	$\sqrt{2}$	$-\sqrt{2}$
$\frac{11\pi}{6}$	$\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{3}$	$-\sqrt{3}$	$\frac{2\sqrt{3}}{3}$	-2