**MA113 – Calculus I** Quiz #10 Name KEY

22 April 2010

This quiz will not be collected for a grade but is intended as a supplement for your study.

Directions: For each of the following questions, **show all your work** in order to justify the necessary steps.

1. Use the Fundamental Theorem of Calculus (Part I) to find for what $x$-value the following function, $F(x)$, achieves its minimum:

$$F(x) = \int_{\frac{11}{4}}^{x} \ln(2t-5)\,dt \qquad \text{for } x > \frac{11}{4}$$

(*Hint:* Use the First Derivative Test)

**Solution:**

Since $f(t) = \ln(2t-5)$ is continuous for $t > \frac{5}{2}$, Part I of the Fundamental Theorem of Calculus shows $F'(x) = f(x) = \ln(2x-5)$. $F'(x) = 0$ if and only if $2x - 5 = 1$, i.e. $x = 3$ (notice the Fundamental Theorem of Calculus ensures $F(x)$ is differentiable everywhere on the interval $\left(\frac{11}{4}, \infty\right)$, so $x = 3$ is the only critical number.) One notices $F'(c) < 0$ for $c \in \left(\frac{11}{4}, 3\right)$ and $F'(d) > 0$ for $d \in (3, \infty)$, so by the First Derivative Test

$$F(3) = \int_{\frac{11}{4}}^{3} \ln(2t-5)\,dt$$

is an absolute minimum.

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2. Find the value of the following sum by recognizing it as a definite integral and use the Fundamental Theorem of Calculus (Part II) to evaluate it exactly:

$$\lim_{n\to\infty} \left( \frac{\pi}{2n} \sum_{k=1}^{n} \cos\left(k \cdot \frac{\pi}{2n}\right) \right)$$

**Solution:**

Comparing the above to the definition of an integral,

$$\int_a^b f(t)\,dt = \lim_{n\to\infty} \sum_{k=1}^{n} f\left(a + k\Delta x\right)\Delta x, \text{ where } \Delta x = \frac{b-a}{n},$$

it is reasonable to assume $f(t) = \cos(t)$ and also that $a = 0$. Since $\Delta x = \frac{\pi}{2n}$, it follows that $b = \frac{\pi}{2}$. This may be verified by noticing $f(a + k\Delta x) = \cos\left(0 + k \cdot \frac{\pi}{2n}\right) = \cos\left(k \cdot \frac{\pi}{2n}\right)$, and therefore the Riemann sum is equivalent to $\int_0^{\frac{\pi}{2}} \cos(t)\,dt$ and since $\cos(t)$ is a continuous function for all real numbers (and in particular on $\left[0, \frac{\pi}{2}\right]$), by Part II of the Fundamental Theorem of Calculus,

$$\int_0^{\frac{\pi}{2}} \cos(t)\,dt = \sin\left(\frac{\pi}{2}\right) - \sin(0) = 1 - 0 = 1.$$