Quiz # 3 for MA 113 - Calculus I

2/04/2010

This quiz is intended to help you prepare for the exams. Thus, you should attempt all questions and write their answers (including your explanations) in the space provided.

This quiz will not be collected or graded.

1. Let f be a function such that, for all real numbers x near 5,

$$\frac{x}{5} + \frac{5}{x} + 2 \leq f(x) \leq x^2 - 10x + 29$$

Argue that $\lim_{x\to 5} f(x)$ exists and find its value.

Solution:

Note that rational and polynomial functions are continuous on their domains. So,

$$\lim_{x\to 5}\left(\frac{x}{5} + \frac{5}{x} + 2\right) = \frac{5}{5} + \frac{5}{5} + 2 = 4$$

and

$$\lim_{x\to 5}\left(x^2 - 10x + 29\right) = 25 - 50 + 29 = 4\ .$$

Because both limits are the same, by the Squeeze Theorem

$$\lim_{x\to 5} f(x) = 4\ .$$

2. Let f be a function defined as

$$f(x) = \begin{cases} x^2 + \sqrt{2x-6}, & for\ x \geq 5; \\ x^2 + x - 3, & for\ x < 5. \end{cases}$$

Show that the function is continuous at $x = 5$.

Solution: Now $f(5) = 5^2 + \sqrt{2(5)-6} = 27$. Note that

$$\lim_{x\to 5^+} f(x) = \lim_{x\to 5^+}\left(x^2 + \sqrt{2x-6}\right) = 5^2 + \sqrt{2(5)-6} = 27$$

because the function is continuous at 5 and

$$\lim_{x\to 5^-} f(x) = \lim_{x\to 5^-}\left(x^2 + x - 3\right) = 5^2 + 5 - 3 = 27$$

because the function is continuous at 5.

This means $\lim_{x\to 5} f(x)$ exists and

$$\lim_{x\to 5} f(x) = 27 = f(5)$$

and f is continuous at 5