MA113 – Calculus I Quiz #5 Solutions

25 February 2010

This quiz will not be collected for a grade but is intended as a supplement for your study.

Directions: For each of the following questions, **show all your work** in order to justify the necessary steps.

1. Simplify each of the following expressions:

   (a) $\arccos(\cos(\frac{7\pi}{4}))$
   Solution:
   Since the range of $\arccos(x)$ is $\{x \,|\, 0 \le x \le \pi\}$, $\cos(\frac{7\pi}{4}) = \cos(\frac{\pi}{4})$ and $\frac{\pi}{4} \in [0, \pi]$,

   $$\arccos\left(\cos\left(\frac{7\pi}{4}\right)\right) = \frac{\pi}{4}.$$

   (b) $\sin(\arctan(-\frac{1}{\sqrt{3}}))$
   Solution:
   Since the range of $\arctan(x)$ is $\{x \,|\, -\frac{\pi}{2} < x < \frac{\pi}{2}\}$ we seek the angle $\theta \in (-\frac{\pi}{2}, \frac{\pi}{2})$ such that $\tan(\theta) = -\frac{1}{\sqrt{3}}$, which implies $\theta = -\frac{\pi}{6}$, and thus

   $$\sin\left(\arctan\left(-\frac{1}{\sqrt{3}}\right)\right) = \sin\left(-\frac{\pi}{6}\right) = -\frac{1}{2}.$$

2. If the position of a particle is given by the following function, find at which time in the given interval the particle is at rest:

   $$s(t) = -\cos^2(t) + t \qquad \text{with } 2\pi \le t \le 4\pi.$$

   [*Hint:* Recall that $\sin(2t) = 2\sin(t)\cos(t)$.]
   Solution:
   To find where the particle is at rest, we must find where the velocity (i.e. the derivative of the position function) is equal to zero in the above interval. Applying the chain rule for $s(t) = f(g(t))$ with $f(t) = t^2$ and $g(t) = \cos(t)$, we obtain

   $$\begin{aligned} v(t) &= s'(t) \\ &= -2(\cos^1(t))(-\sin(t)) + 1 \\ &= 2\cos(t)\sin(t) + 1 \\ &= \sin(2t) + 1. \end{aligned}$$

   Now we let $v(t) = 0$ so that $\sin(2t) + 1 = 0$, which is true if and only if $\sin(2t) = -1$, which implies $2t = \frac{3\pi}{2}$. So $t = \frac{3\pi}{4}$ and since the period of $\sin(2t)$ is π and $\sin(\frac{3\pi}{4}) = \sin(\frac{11\pi}{4})$, $\frac{11\pi}{4}$ is in the desired interval, as is $\frac{11\pi}{4} + \pi = \frac{15\pi}{4}$. Therefore

   $$v(t) = 0 \text{ for } 2\pi \le t \le 4\pi \text{ when } t = \frac{11\pi}{4} \text{ and } \frac{15\pi}{4}.$$