Quiz # 6 for MA 113 - Calculus I

4 March 2010

This quiz is intended to help you prepare for the exams. Thus, you should attempt all questions and write their answers (including your explanations) in the space provided.

This quiz will not be collected or graded.

1. (Modified from Stewart Calculus, Section 3.8, Problem 10)
A sample of unobtainium decayed to 83.2% of its original amount after one second.
a) What is the half-life of unobtainium?
b) How long would it take the sample to decay to 20% of its original amount?
Solution:
a) Let m_0 be the amount of unobtainium at time $t = 0$. We know that decay is given by the function $f(t) = m_0e^{kt}$ where k is some constant.
We are given that $f(1) = 0.832m_0$, so substitute 1 for t. $0.832m_0 = f(1) = m_0e^k$
Solve for k. $k = \ln(0.832)$
Substitute $k = \ln(0.832)$ into $f(t)$, and set equal to $0.5m_0$. $0.5m_0 = m_0e^{t\ln(0.832)} = m_0 0.832^t$
Solve for t. $t = \log_{0.832}(0.5) = \frac{\ln(0.5)}{\ln(0.832)}$
The half-life of unobtainium is $\frac{\ln(0.5)}{\ln(0.832)} \approx 3.769$ seconds.
b)
Set $f(t)$ equal to $0.2m_0$. $0.2m_0 = m_0 0.832^t$
Solve for t. $t = \log_{0.832}(0.2) = \frac{\ln(0.2)}{\ln(0.832)}$
The sample of unobtainium will decay to 20% of its original mass after $\frac{\ln(0.2)}{\ln(0.832)} \approx 8.751$ seconds.

2. (Modified from Exam 2, Spring 2009, Problem 10)
A small boat is being pulled into a dock that is 2 feet above the water surface by a rope attached to the bow and going up over a pulley located at the end of the dock. The rope is attached to the boat at a point that is 2 feet above the surface of the water and the pulley is 3 feet above the dock. The rope is being reeled in at a rate of 2 feet per second. How fast is the boat coming towards the dock at the moment the bow is 4 feet away from the base of the pulley stand? (Recall that $3^2 + 4^2 = 5^2$.)

Solution:
Let $y = y(t)$ be the length of the rope between the boat and the pulley at time t and let $x = x(t)$ be the distance between the boat and the dock at time t. We are given that $y'(t) = -2\frac{ft}{s}$.
By the Pythagorean Theorem we have $[y(t)]^2 = [x(t)]^2 + 3^2$.
Differentiating both sides with respect to t we get $2y(t)y'(t) = 2x(t)x'(t)$.
Let $t = t_0$ be the moment that the boat is 4 feet away from the dock. That is $x(t_0) = 4$.
Then, $[y(t_0)]^2 = 4^2 + 3^2 = 5^2$. So, $y(t_0) = 5$.
Then at $t = t_0$ we have $(2)(5)(-2) = (2)(4)x'(t_0)$. So, $x'(t_0) = \frac{(2)(5)(-2)}{(2)(4)} = \frac{-20}{8} = \frac{-5}{2}$.

So, at the moment the bow is 4 feet away from the base of the pulley, the boat is coming towards the dock at a rate of $\frac{5}{2}\frac{ft}{s}$.