Quiz # 7 [SOLUTION] for MA 113 - Calculus I

March 25, 2010

This quiz is intended to help you prepare for the exams. Thus, you should attempt all questions and write your answers (including your explanations) in the space provided.

This quiz will not be collected or graded.

1. Consider the function $f(x) = \frac{x^4}{4} + \frac{x^3}{3} - x^2 - \frac{2}{3}$.
   (a) Find all the critical numbers of f.
   (b) Find the interval(s) where f is increasing.
   (c) Find the interval(s) where f is decreasing.
   (d) Find the local maximum and local minimum.

   Answer:

   (a) Since f is differentiable on $(-\infty, \infty)$, then all critical numbers of f occur when $f'(x) = 0$. Since
   $$f'(x) = x^3 + x^2 - 2x = x(x^2 + x - 2) = x(x+2)(x-1),$$
   then $f'(x) = 0$ when $x = -2, 0, 1$. Thus, the critical numbers of f are $-2, 0$, and 1.
   (b) f is increasing when $f'(x) > 0$. Thus, f is increasing on $(-2, 0)$ and $(1, \infty)$.
   (c) f is decreasing when $f'(x) < 0$. Thus, f is decreasing on $(-\infty, -2)$ and $(0, 1)$.
   (d) Local maximums and minimums occur when $f'(x) = 0$. Thus, the local maximums and minimums must occur when $x = -2, 0$, or 1. Since $f'(x)$ changes from negative to positive at $x = -2$ and $x = 1$, the local minimums are $f(-2) = -\frac{10}{3}$ and $f(1) = -\frac{13}{12}$. Since $f'(x)$ changes from positive to negative at $x = 0$, the local max is $f(0) = -\frac{2}{3}$.

2. Let g be a continuous and differentiable function. Let $g(2) = 5$. If $g'(x) \geq 3$, then use the Mean Value Theorem to determine the smallest possible value of $g(8)$?

   Answer: Since g is continuous and differentiable on $(-\infty, \infty)$, the MVT guarantees the existence of some x such that
   $$g'(x) = \frac{g(8) - g(2)}{8 - 2} = \frac{g(8) - 5}{8 - 2} = \frac{g(8) - 5}{6}.$$
   Since $g'(x) \geq 3$ for all x, then
   $$\frac{g(8) - 5}{6} \geq 3.$$
   Thus,
   $$g(8) - 5 \geq 18.$$
   Therefore, 23 is the smallest possible value of $g(8)$.