Quiz # 8 for MA 113 - Calculus I

4/01/2010

This quiz is intended to help you prepare for the exams. Thus, you should attempt all questions and write their answers (including your explanations) in the space provided.

This quiz will not be collected or graded.

1. Assume that f and g are differentiable functions on $(-\infty, \infty)$ and $f'(x) = g'(x)$ for all x. Let $f'(x) = \cos x$ and $g(\frac{\pi}{2}) = -1$. Find $g(x)$.

Solution:

As a consequence of the Mean Value Theorem, if $f'(x) = g'(x)$ for all x then f and g differ by at most a constant c. One function $f(x)$ such that $f'(x) = \cos x$ is given by $f(x) = \sin x$. So $g(x) = \sin x + c$ for some constant c. Note that $-1 = g(\frac{\pi}{2}) = \sin \frac{\pi}{2} + c = 1 + c$. So, $c = -2$. Thus, $g(x) = \sin x - 2$.

2. Use l'Hospital's rule to evaluate the following limits:

1. $\lim_{x \to 0} \frac{\tan(\pi x)}{x}$

2. $\lim_{x \to \infty} e^{-x} \ln(x + 3)$

Solution:

1. This has type $\frac{0}{0}$ indeterminant form. Thus

$$\lim_{x \to 0} \frac{\tan(\pi x)}{x} = \lim_{x \to 0} \frac{\frac{d}{dx} \tan(\pi x)}{\frac{d}{dx} x} = \lim_{x \to 0} \frac{\pi \cdot \sec^2(\pi x)}{1} = \pi \cdot \sec^2(0) = \pi \ .$$

2. This has type $0 \cdot \infty$ indeterminant form. So we write

$$e^{-x} \ln(x + 3) = \frac{\ln(x + 3)}{\frac{1}{e^{-x}}} = \frac{\ln(x + 3)}{e^x} \ .$$

Note that

$$\lim_{x \to \infty} \frac{\ln(x + 3)}{e^x}$$

is of type $\frac{\infty}{\infty}$.

Thus

$$\lim_{x \to \infty} e^{-x} \ln(x + 3) = \lim_{x \to \infty} \frac{\ln(x + 3)}{e^x} = \lim_{x \to \infty} \frac{\frac{1}{x+3}}{e^x} = \lim_{x \to \infty} \frac{1}{e^x (x + 3)} = 0 \ .$$