Quiz # 9 for MA 113 - Calculus I

04/8/2010

This quiz is intended to help you prepare for the exams. Thus, you should attempt all questions and write their answers (including your explanations) in the space provided.

This quiz will not be collected or graded.

1. A square box with an open top is to have a volume of 4000 cm^3. Find the dimensions of the box that minimizes the amount of material used.

Let x be the length of the sides of the square bottom and let y be the height of the box. Since the volume of the box is 4000 cm^3, we must have that

$$x^2 y = 4000 \Rightarrow y = \frac{4000}{x^2}$$

Since we want to minimize the material used, we use the following equation for the material used:

$$M = x^2 + 4yx$$

Substituting for y gives us

$$M = x^2 + \frac{16000}{x}$$

Next, we take the derivative to get

$$M' = 2x - \frac{16000}{x^2}$$

To find a possible minimum we set this equation equal to 0 and solve.

$$0 = 2x - \frac{16000}{x^2} \Rightarrow \frac{16000}{x^2} = 2x \Rightarrow 8000 = x^3 \Rightarrow x = 20$$

We then check that this is a minimum by using the first derivative test for absolute minima. $M'(19) < 0$ and $M'(21) > 0$. Lastly, we then use x to solve for y.

$$(20)^2 y = 4000 \Rightarrow y = 10$$

Thus the dimensions for such a box using the least amount of material is 20cm×20cm×10cm.

2. Use linear approximation to estimate the value of $\sqrt{4.1}$.

We first observe that $4.1 \approx 4$. Next, if we let

$$f(x) = \sqrt{x},$$

then

$$f'(x) = \frac{1}{2\sqrt{x}}$$

Using this information we can compute the linearization of $f(x)$ at $x = 4$, obtaining

$$L(x) = \frac{1}{2\sqrt{4}}(x - 4) + \sqrt{4} = \frac{1}{4}(x - 4) + 2$$

Evaluating the linearization at x=4.1, we obtain our approximation.

$$L(4.1) = \frac{1}{4}(4.1 - 4) + 2 = 2.025$$