MA671–001 Complex Analysis
Spring 2020
Problem Set 3
DUE: Monday, 24 February 2020

1. Harmonic functions and conjugates

   (a) Stein and Shakarchi, page 27: Problem 11.

   (b) If u on a simply connected set $\Omega \subset C$ (like a disk) is harmonic, then there exists a harmonic function v on ω so that $f = u + iv$ is analytic on Ω. (Such a function v is called a *harmonic conjugate* of u.)

2. Stein and Shakarchi, page 65: Problem 6.

3. Stein and Shakarchi, page 65: Problem 7.

4. Prove the Mean Value Property for harmonic functions: Suppose u is harmonic on a region containing a circle of radius $r_0 > 0$ about z_0. Then:

$$u(z_0) = \frac{1}{2\pi} \int_0^{2\pi} u(z_0 + r_0 e^{i\theta} \, d\theta.$$

5. Compute the Taylor series of $f(z) = (z^2 + 4)^{-1}$ about $z = 0$. What is the radius of convergence?