MA681–001 Functional Analysis
Fall 2013
Problem Set 1
DUE: Wednesday, 11 September 2013

1. Show that $C([0,1])$ with the sup-norm is a Banach space. Let A be the set of all polynomials with rational complex coefficients (that is, those of the form $a + ib$ with $a, b \in Q$). Show that A is dense in $C([0,1])$. Conclude that this Banach space is separable.

2. Banach Algebras. A *Banach algebra* is a Banach space that is also an algebra. This means that there is a multiplication of elements which is compatible with all the other structures. In particular, if $x, y \in A$, then $xy \in A$ and $\|xy\| \leq \|x\| \, \|y\|$, and the multiplication is distributive. Show that $M_n(C)$ and $(C([0,1]), \|\cdot\|_\infty)$ are Banach algebras.

3. Consider the Banach space $(C([0,1]), \|\cdot\|_\infty)$. Let $a \in C([0,1])$ and define the multiplication operator $A : f \in C([0,1]) \to af$. Show that A is a bounded linear operator and compute it's norm. If $|a| < 1$, show that the operator $1 + A$ is a boundedly invertible operator.