MA681–001 Functional Analysis
Fall 2013
Problem Set 5
DUE: Wednesday, 20 November 2013

(1) Problem 5.4 on page 53.
(2) Problem 7.4 on page 73.
(3) Prove that if $A : D(A) \subset \mathcal{H} \to \mathcal{H}$ is a closed operator and a bijection then A^{-1} is bounded.
(4) Let $A \in \mathcal{L}(\mathcal{H})$ and for f analytic in a neighborhood of $\sigma(A)$, let $f(A)$ be the integral
$$f(A) = \frac{-1}{2\pi i} \int_\gamma f(z) R_A(z)\ dz,$$
where γ is a simple closed contour around $\sigma(A)$ with counter clockwise orientation. Show that $f(A)$ is a bounded operator. If g is another such function, show that $(fg)(A) = f(A)g(A)$.
(5) Continuing the preceding problem, suppose $\sigma(A) = \sigma_1(A) \cup \sigma_2(A)$ is a disjoint decomposition of the spectrum into two nonempty sets. Let γ be simple closed contour encircling $\sigma_1(A)$ once in the counter clockwise sense and having no intersection with $\sigma_2(A)$. Show that the operator
$$P_1 = \frac{-1}{2\pi i} \int_\gamma R_A(z)\ dz,$$
is a projection operator, that $AP = PA$, and that the spectrum of AP is $\sigma_1(A)$.