Answers Test 1 **MA 433 S08**

1. (a) $\{4e^{-i\pi/6}, 4e^{i\pi/2}, 4e^{i7\pi/6}\} = \{2\sqrt{3}-2i, 4i, -2\sqrt{3}-2i\}$.

(b) $\lim\limits_{z\longrightarrow -1} \frac{(z^3+2)^4-1}{z+1} = \frac{d}{dz}(z^3+2)^4\,|_{z=-1} = 12z^2(z^3+2)^3\,|_{z=-1} = 12$.

(c) $\dfrac{1+e^{i\pi/3}}{1-e^{i\pi/3}} = \dfrac{(1+e^{i\pi/3})(1-e^{-i\pi/3})}{|1-e^{i\pi/3}|^2} = i\sqrt{3}$. Thus, Re $\dfrac{1+e^{i\pi/3}}{1-e^{i\pi/3}} = 0$.

(*d*) Ω is not bounded,
but open and connected.

.

2. $w = \rho e^{i\phi}$ and $z = re^{i\theta}$, then $\rho e^{i\phi} = r^4 e^{4i\theta}$. Thus $|w| = \rho = r^4 = |z|^4$. and 4θ is in arg w. It follows that f maps circles into circles and rays into rays. Using this fact and one to one ness of f it can be shown that $f(\partial S) = \{z : |z| = 1\} \cup \{z : |z| = 16\} \cup [-16, -1]$.
(b) $f(S)$ is the domain bounded by the circles $|w| = 1, 16$ and the negative real axis. That is the inside of the curve in (a).

3. Multiply out to get $f(z) = (y^3 - 3x^2y + 2x - 1) + i(x^3 - 3xy^2 + 2y)$. Thus $u(x, y) = y^3 - 3x^2y + 2x - 1$ and $v(x, y) = x^3 - 3xy^2 + 2y$. (a) $u_x = -6xy + 2 = v_y$ and $u_y = 3y^2 - 3x^2 = -v_x$.
(b) At (x, y), $\Delta v = v_{xx} + v_{yy} = 6x - 6x = 0$. Since v has continuous second partials everywhere, it follows that v is harmonic for all (x, y).

4. (a) $g_r = (1/3)r^{-2/3}e^{i\theta/3}$ and $g_\theta = (i/3)r^{1/3}e^{i\theta/3}$. Thus $g_r = -ir^{-1}g_\theta$ whenever $r > 0, |\theta| < \pi$ or when z is not in $(-\infty, 0]$.
(b) By class work, $g'(z)$ exists at z if g has continuous first partials and the CR equations hold at z. Clearly g has continuous second partials in r, θ when $r > 0, |\theta| < \pi$. Using (a) it follows that $g'(z)$ exists when z is not in $(-\infty, 0]$.
(c) $8i = 8e^{i\pi/2}$ so by classwork $f'(z) = e^{-i\pi/2}f_r$ (evaluated at $r = 8$, $\theta = \pi/2$) $= (1/3)8^{-2/3}e^{-i\pi/3} = (1/24)(1-\sqrt{3}i)$.
(d) $O = \{z = x + iy : x \neq 0\}$, since z^2 maps $\{z = x + iy : x \neq 0\}$ onto $\mathbf{C} \setminus (-\infty, \mathbf{0}]$ and points on the imaginary axis are mapped into nonpositive numbers.

EC. Want to find v so that $v_y = u_x = 5e^{5x+3}\sin(5y) + y$. Thus,
$v(x, y) = \int v_y dy = -e^{5x+3}\cos(5y) + y^2/2 + h(x)$. From the Cauchy Riemann equations, want $v_x = -u_y = -5e^{5x+3}\cos(5y) - x + 4$. From the expression for v, $v_x = -5e^{5x+3}\cos(5y) + h'(x)$. Equating, the two equations gives, $h'(x) = -x + 4$. Integrate with respect to x, and get $h(x) = -x^2/2 + 4x + c$. Thus $v(x, y) = -e^{5x+3}\cos(5y) + (y^2 - x^2)/2 + 4x + c$.
(b) $f = u + iv = -i(e^{5z+3} + z^2/2 - 4z)$.

Homework Problems

Section 3.28 (page 89) 1 (b), 4, 8 (a),(c), 13.
Section 3.30 (page 94) 1 (b), 2 (a),(c), 3 (b), 7, 9 (a), 10.
Section 3.31 (page 96) 1, 6.
Section 3.32 (page 99) 1, 2 (a),(c), 3.
Section 3.33 (page 103) 2, 11, 18.
Section 3.34 (page 107) 1, 7 (b), 15 (a).
Section 3.35 (page 110) 1 (a), 2 (b), 4.
Section 4.37 (page 115) 2 (a), (c), 3, 7.
Section 4.40 (page 129) 1 (a), (c), 3, 7, 10.
Section 4.41 (page 133) 2, 4, 5.
Section 4.43 (page 141) 2, 3, 5.
Section 4.46 (page 153) 1 (a), (e), (f), 3.
Section 4.49 (page 162) 1 (b), (c), 2 (b), 3.
Section 4.50 (page 171) 1, 7, 8.
Section 5.52 (page 181) 1, 4.
Section 5.54 (page 189) 2, 5, 7, 11 (b).
Section 5.56 (page 198) 1, 4, 6, 10 (a).
Section 5.60 (page 212) 1, 6, 7.
Section 5.60 (page 218) 1, 2, 8.