

The test is over sections 7.4, 7.7, 7.8, in chapter 7 and sections 11.1 - 11.5 in chapter 11. Topics include,

1. Integration Techniques: integration by decomposing into partial fractions.
2. Improper Integrals: Infinite limits of integration, discontinuous integrands, comparison test for convergence of improper integrals.
3. Approximate Integration: Midpoint, Trapezoidal, and Simpson's rule, error bounds.
4. Sequences and Series: Convergence of a sequence and series, geometric and telescoping series.
5. Convergence tests for positive term series: integral, comparison and limit comparison test.
6. Other Convergence tests: alternating series test (including error estimates for partial sums) and the test for divergence.

As usual going over the quizzes, suggested homework problems, class and recitation notes, should be a good review for the test. Suggested review problems from Chapter 7, Page 518 are, Concept Check: 4 - 8, True-False Quiz: Odds 1-13, Exercises, 15,19,43,47,63,65,71. Suggested review problems from chapter 11, Page 758, are Concept Check: 1,3,5 (a)-(e), True - False Quiz: 1,3,11,15. Exercises: 1,7,11,15,21,23,27,29.

Homework Problems from which homework quizzes will be taken after the test follow:

- Section 11.6 (page 719) (possible HW quiz questions) 1,3,7,13,21,23,27.
- Section 11.6 (page 719) (additional problems) 5,9,15,19,25,29.
- Section 11.8 (page 727) (possible HW quiz questions) 5,9,15,19,23,27.
- Section 11.8 (page 727) (additional problems) 3,11,17,21,25.
- Section 11.9 (page 733) (possible HW quiz questions) 5,9,13 (a),15,23,27.
- Section 11.9 (page 733) (additional problems) 3,11,17,21,25,31.
- Section 11.10 (page 746) (possible HW quiz questions) 7,13,19,21,25,33,47,51,55,61.
- Section 11.10 (page 746) (additional problems) 11,15,17,23,27,35,41,49,57,59,67.
- Section 8.1 (page 530) (possible HW quiz questions) 3,5,7,11,15.
- Section 8.1 (page 530) (additional problems) 1,9,13,17.
- Section 10.1 (page 626) (possible HW quiz questions) 1,7,9,11,15,21.
- Section 10.1 (page 626) (additional problems) 3,5,13,19,13,19,37,43.
- Section 10.2 (page 636) (possible HW quiz questions) 3,7,19,27,41,43,45.
- Section 10.2 (page 636) (additional problems) 5,9,17,29,39,47,53.
- Section 10.3 (page 647) (possible HW quiz questions) 1,3,5,9,17,25,33,37,43,59.
- Section 10.3 (page 647) (additional problems) 7,11,15,19,21,27,29,35,39,47,57,63.
- Section 10.4 (page 653) (possible HW quiz questions) 1,7,11,17,27,29,33,47,53.
- Section 10.4 (page 653) (additional problems) 5,9,15,19,23,25,31,35,45.
- Section 10.5 (page 660) (possible HW quiz questions) 7,15,21,27,33,39,45.
- Section 10.5 (page 660) (additional problems) 5,9,17,19,23,29,35,43.
- Section 10.6 (page 668) (possible HW quiz questions) 3,5,11,15.
- Section 10.6 (page 668) (additional problems) 1,9,13,21.

Here are two practice tests for Test 2.

Practice Test 1

25 points 1. (a) Find the partial fraction decomposition of $\frac{3x - 1}{x^2 + 2x - 15}$.

(b) Use (a) to find $\int \frac{3x - 1}{x^2 + 2x - 15} dx$.

25 points 2. A theorem in the book states that if M_n is the n th midpoint approximation to $I = \int_a^b f(x) dx$, then $|I - M_n| \leq K(b - a)^3/24n^2$. Use this theorem to determine how large n has to be to approximate $I = \int_0^{\pi/2} \sin(2x) dx$ by M_n within $\frac{1}{1000}$.

25 pts 3. Find the following limits and/or sums if they exist. Show your work! Credit will not be given for just a calculator answer.

(a) $\lim_{n \rightarrow \infty} n^{-1/2} (\ln(n^5))$

(b) $\lim_{n \rightarrow \infty} \sum_{k=1}^n 2^{-k/2}$

(c) $\lim_{n \rightarrow \infty} \sum_{k=1}^n \arctan(n)$

25 pts 4. Given that the p series, $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges for $p > 1$ and diverges for $p \leq 1$.

Determine whether the following series converge. In each case indicate which convergence tests you are using. Make sure you show each series satisfies the conditions of the test you are using.

(a) $\sum_{n=1}^{\infty} ne^{-n^2}$

(b) $\sum_{n=1}^{\infty} \sqrt{n+1}/(n+3)^2$

(c) $\sum_{n=1}^{\infty} (-1)^n \frac{1}{n^{1/4}}$

10 pts EC. Find the following improper integral. Include limiting arguments. $\int_{-1}^0 (1 - x^2)^{-1/2} dx$.

Answers Practice Test 1

1. (a) $\frac{1}{x - 3} + \frac{2}{x + 5}$

(b) $\ln|x - 3| + 2 \ln|x + 5| + c$.

2. $n \geq 5\pi\sqrt{5\pi/6}$.

3. (a) 0.

(b) $(\sqrt{2} - 1)^{-1}$.

(c) No limit as the series diverges by the test for Divergence.

4. (a) Converges by either the integral, comparison, or limit comparison tests.
 (b) Converges by the limit comparison test and comparison with a p series when $p = 3/2$.
 (c) Converges by the alternating series test.

EC. $\pi/2$.

Practice Test 2

30 pts 1. Determine whether the following integrals converge. Give all limiting arguments.

(a) $\int_{-\infty}^{\infty} \frac{x}{1+x^2} dx$

(b) $\int_{-1}^0 \frac{1}{\sqrt{x+1}} dx$.

20 pts 2. Given Simpson's rule :

$$\int_a^b f(x) dx \approx (\Delta x/3) (f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \dots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)),$$

where n is even and $\Delta x = (b - a)/n$. Use this rule with $n = 6$ to find a good approximation to

$$\int_0^3 \frac{1}{1+x^3} dx.$$

20 pts 3. (a) State the integral test for convergence of a positive term series.

(b) Use this test to determine whether the series $\sum_{n=1}^{\infty} \frac{1}{n^2 + 9}$ converges or diverges.

30 pts 4. Determine whether the following series and/or sequences converge. If possible find the sum of the series and/or limit of the sequence. You may assume that $\sum_{n=1}^{\infty} n^{-p}$ converges for $p > 1$ and diverges for $p \leq 1$. Indicate your reasoning.

(a) $\lim_{n \rightarrow \infty} \frac{\cos(\pi n) + e^{-n}}{2 + 5/n}$

(b) $\sum_{n=2}^{\infty} \frac{3}{(n+4)(n+5)}$

(c) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{(\ln(n+3))^2}$.

(d) $\sum_{n=1}^{\infty} \frac{e^n}{n^2}$

10 pts EC. Write down the general form of the rational decomposition of each of the rational expressions in (a) and (b). Do not find the coefficients of these expansions.

For example, $\frac{1}{x^2-1} = \frac{A}{x-1} + \frac{B}{x+1}$.

(a) $\frac{x+6}{(x^2+x+1)^2}$

(b) $\frac{x^2+3}{(x+4)^2(x+5)^2}$

5 pts EEC. Given that $\ln 2 = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n}$. Let $S_k = \sum_{n=1}^k \frac{(-1)^{n-1}}{n}$ be the k th partial sum of this series. Use the error estimate in the alternating series test to find an estimate for $|\ln 2 - S_{150}|$.

Answers Practice Test 2

1. (a) diverges,
 (b) 2.

2. $(.5/3) \left[1 + 4\frac{1}{1+(.5)^3} + 2\frac{1}{2} + 4\frac{1}{1+(1.5)^3} + 2\frac{1}{1+2^3} + 4\frac{1}{1+(2.5)^3} + \frac{1}{1+3^3} \right] \approx 1.16232$

3. See the book, section 11.3. Given series converges.

4. (a) Limit does not exist.
 (b) $1/2$
 (c) Converges by the alternating series test
 (d) Diverges by the test for Divergence.

EC. (a) $\frac{Ax + B}{(x^2 + x + 1)^2} + \frac{Cx + D}{x^2 + x + 1}$

(b) $\frac{A}{(x + 4)^2} + \frac{B}{x + 4} + \frac{C}{(x + 5)^2} + \frac{D}{x + 5}.$

EEC. $|\ln 2 - S_{150}| \leq 1/151.$