UNIVERSITY OF KENTUCKY
COLLEGE OF ARTS AND SCIENCES
DEPARTMENT OF MATHEMATICS
Professor John Lewis (859) 257-1153
765 Patterson Office Tower john@ms.uky.edu
Lexington, Kentucky 40506-0027 FAX No. (606) 257-4078

October 17, 2009

Dear Colleague

I am writing this letter on behalf of Professor Katharine Ott who I understand is applying for an assistant professor position in your department. Katharine is a former PhD student of Professor Irina Mitrea at the University of Virginia. Her thesis and subsequent papers are concerned with boundary value problems on Lipschitz domains. This topic has been very popular over the last 35 years with important constributions from such well known mathematicans as Calderón, David, Kenig, Jerison, Jones, Pipher, and Verchota. This area is still thriving today with the Mitrea trio of Dorina, Irina, and Marius leading the way in their research on transmission, electromagnetic scattering, the homogeneous - inhomogeneous mixed and spectral radius problems in Lipschitz domains. I am familiar with the following of Professor Ott's papers.

[1] *Counterexamples to the well-posedness of L^p transmission boundary value problems for the Laplacian* (with Irina Mitrea), Proceedings of the American Mathematical Society, **135** (2007), 2037-2043.

[2] *Electromagnetic scattering from perturbed surfaces* (with Irina Mitrea), Mathematical Methods in the Applied Sciences **30** (2007), 861-876.

[3] *Spectral theory and iterative methods for the Maxwell system in non smooth domains* (with Irina Mitrea), submitted.

[4] *The mixed problem for the Laplacian in Lipschitz domains* (with R. Brown), submitted.

In order to discuss these papers let Ω be a Lipschitz domain and put $\Omega_- = \mathbb{R}^n \setminus \bar{\Omega}, \Omega_+ = \Omega$. In [1] Katharine and Irina consider the transmission problem

$$\begin{cases} \Delta u_\pm = 0 \text{ in } \Omega_\pm \\ N^*(\nabla u \pm |) \in L^p(\partial\Omega) \\ u_+|_{\partial\Omega} = u_-|_{\partial\Omega} \\ \partial_\nu u_+ - \gamma \partial_\nu u_- = g \in L^p(\partial\Omega) \end{cases}$$

where $\gamma \in (0,1)$ is the transmission coeffient, N^* is the nontangential maximal function operator, and ∂_ν is the outer normal derivative to $\partial\Omega$ relative to Ω. The authors point out that work of Spencer and Shelepov can be used to show for any $p > 2$ that there exists a sector $\Omega \subset I\!\!R^2$ for which the above boundary value problem does not have a unique solution. Their example complements work of Marius Mitrea and Luis Ecauriaza who showed that this problem always has a unique solution in a Lipschitz domain when $1 < p \leq 2$. In [2] and [3] Professors Mitrea and Ott consider the following boundary value problem for the electric field $\vec{E} = (E_1, E_2, E_3)$:

$$\begin{cases} (\Delta + k^2)\vec{E} = 0 \text{ in } I\!\!R^3 \setminus \bar{\Omega} \\ \nabla \cdot \vec{E} = 0 \text{ in } I\!\!R^3 \setminus \bar{\Omega} \\ \nu \times \vec{E}|_{\partial\Omega} = \vec{g} \in L^2{}_{\text{tan}}\ (\partial\Omega) \\ \vec{E} \text{ satisfies the Silver - Muller radiation condition at } \infty. \end{cases}$$

In [2] they consider the effect on this boundary value problem when certain bumps are added to $\partial\Omega$. They show the above boundary value problem on the perturbed surface can be reduced to solving a certain integral equation on the bumps (assuming knowledge of the Green's function of the unperturbed domain). In [3] they use Rellich type identities to study the spectrum of the magnetic dipole singular integral operator, $\tilde{M}_k$, associated with the above boundary value problem. They show that if $\partial\Omega \subset I\!\!R^2$ has Lipschitz norm $\leq \omega$, then $zI + \tilde{M}_k$ is semi Fredholm with index zero on $L^2(\partial\Omega)$ whenever z lies in the interior of the hyperbola with vertices $(\pm\frac{\omega}{\sqrt{\omega^2+1}}, 0)$ and asymptopes having slopes $\pm 1/\omega$. Finally in [4] Professors Brown and Ott study the mixed problem when $\partial\Omega = D \cup N, N \cap D = \emptyset$.

$$\begin{cases} \Delta u = 0 \text{ in } \Omega \\ u = f_D \text{ on } D \\ u = f_N \text{ on } N. \end{cases}$$

They show that there exists $q_0 > 1$ such that if $1 < p < q_0$ and $f_N \in L^p(N), f_D \in W^{1,p}(D)$, then the above mixed problem always has a solution u with

$$\|N^*(|\nabla u|)\|_{L^p(\partial\Omega)} \leq c\,(\|f_N\|_{L^p(N)} + \|f_D\|_{W^{1,p}(D)}).$$

This paper represents somewhat of a breakthrough on the mixed problem as earlier papers required either a geometric assumption on the interface between D, N or were limited to $I\!\!R^2$. The key argument involves using a technique of Shen to first prove a reverse Hölder inequality for certain solutions to the mixed problem and second use a good bad λ argument to get the desired integrability.

It is clear from her papers that Professor Ott is rapidly developing into a strong harmonic analysis - partial differential equations researcher with comand of an impressive armada of techniques. In the future I expect that she will continue to widen her research areas and also continue to develop her already extensive knowledge of singular integrals. Rellich inequalities, and scattering theory. Finally, Katharine is a very friendly person who seems to get along with everyone and whose energy - enthusiasm has been a real plus for our department. Currently she is our PDE seminar chairman. Thus I hope that if you have an opening for a person in harmonic analysis - partial differential equations that you will give Professor Ott strong consideration.

Sincerely,

John Lewis