Name:_____

Test 2
Apportionments and Number Theory
A&S 100
Fall 2002

Objectives.

You should be able to do the following:

1. Apportion the seats of a house using the Hill-Huntington method.
2. Find threshold divisors for the Hill-Huntington methods. Use these threshold divisors to find an appropriate divisor for the Hill-Huntington method.
3. Compare and contrast Hamilton's, Lowndes', Jefferson's, Webster's, and the Hill-Huntington Methods of apportionment. Which methods favor smaller states? Which methods favor larger states?
4. Define the Quota Property, the House Size Property, and The Population Property.
5. Identify the Alabama Paradox and note that it arose from an application of Hamilton's method.
6. Recognize that Jefferson's method is expected to violate the Quota Property more frequently than the Hill-Huntington Method which is expected to violate the Quota Property more frequently than Webster's Method.
7. State why the House Size Property might be irrelevant in determining an apportionment method for the U.S. House of Representatives.
8. State:
   (a) No divisor method satisfies the Quota Property.
   (b) Every divisor method satisfies the House Size Property.
   (c) An apportionment method satisfies the Population Property if and only if it is a type of divisor method in which the rounding depends on the number of states and the house size.
   (d) No apportionment method satisfies both the Quota Property and the Population Property.
9. Recognize integers.
10. For integers a and $b \neq 0$ define:
    (a) b **divides** a, $b|a$

(b) a **is divisible by** b

11. Recognize and use the notation $b|a$ for b divides a.
12. Define **factor, divisor, prime number,** and **composite number.**
13. State the Fundamental Theorem of Arithmetic.
14. Know that there are an infinite number of prime numbers.
15. Be able to argue that there are an infinite number of prime numbers.
16. Apply the Test for Primality to determine if a number is prime or composite.
17. Prove the Test for Primality.
18. State the Division Algorithm.
19. Given integers a and $b \neq 0$, use the Division Algorithm to find the quotient and the remainder when a is divided by b.
20. Given integers a and b, find $\gcd(a, b)$ using:
    (a) the prime factorizations of a and b.
    (b) the Euclidean Algorithm.
21. For integers $n > 0$, a, and b define:
    (a) $a \bmod n$
    (b) **congruence modulo** n, $a \equiv b \pmod{n}$.
22. For integers a, b and n with $n > 0$:
    (a) Find $a \bmod n$.
    (b) Determine if $a \equiv b \pmod{n}$
23. Know and be able to apply the properties of modular arithmetic on page 483 of your text.
24. Know and be able to apply the divisibility tests for $2, 3, 4, 5, 6, 7, 8, 9, 10,$ and 11.
25. Prove the divisibility tests for 2–6 and 8–11.
26. Derive and prove a divisibility test for 2^k.
27. Calculate check digits according to various check digit formulas.
28. Use check digits to detect errors in data.
29. Recognize that the check digit formulas discussed in class do not detect errors with 100% accuracy.