

Name: \_\_\_\_\_

Test 3  
Tilings, Polyhedra, and Game Theory  
A&S 100  
Fall 2002

## **Objectives.**

You should be able to do the following:

1. Demonstrate understanding of the following terms:

- polygon
- $n$ -gon
- side
- vertex (vertices) of a polygon
- convex polygon
- concave polygon
- (interior) angles of a polygon
- regular polygon
- equiangular polygon
- equilateral polygon
- tiling
- tessellation
- monohedral tiling
- edge-to-edge tiling
- vertex of a tiling
- regular tiling
- vertex type of a vertex in a tiling
- semiregular tiling
- periodic tiling
- aperiodic tiling
- polyhedron; polyhedra
- face of a polyhedron
- edge of a polyhedron
- vertex of a polyhedron

- convex polyhedron
  - concave polyhedron
  - Euler's Formula for Convex Polyhedron
  - regular polyhedron; Platonic solids
  - semiregular polyhedron
  - vertex type of a polyhedron
  - prism
  - antiprism
  - Archimedean solids
  - alternative move games
  - move
  - game tree
  - partial game tree
  - strategy
  - optimal strategy
  - compressed game tree
  - partial compressed game tree
2. Understand the relationship between convexity and concavity and the interior angles of a polygon.
  3. Recognize that the common name for a regular triangle is "equilateral triangle."
  4. Recognize that the common name for a regular quadrilateral is "square."
  5. For  $n \geq 4$ , know that there are equilateral  $n$ -gons which are not regular. Provide an example of such an  $n$ -gon.
  6. For  $n \geq 4$ , know that there are equiangular  $n$ -gons which are not regular. Provide an example of such an  $n$ -gon.
  7. Know that the sum of the interior angles of a triangle is  $180^\circ$  and use this fact to argue that the sum of the interior angles of a convex  $n$ -gon is  $(n - 2)180^\circ$ .
  8. For a given  $n$ -gon, use triangles to show that the sum of the interior angles is  $(n - 2)180^\circ$ .
  9. Know that the sum of the interior angles of any  $n$ -gon is  $(n - 2)180^\circ$ .
  10. Use the formula for the sum of the interior angles of an  $n$ -gon to derive the formula for the measure of an interior angle of a regular  $n$ -gon.
  11. Know that the only regular tilings use equilateral triangles, squares, or regular hexagons.

12. Prove that the only regular tilings use equilateral triangles, squares, or regular hexagons.
13. Know that there are only 8 semiregular tilings.
14. Provide an example of regular polygons which fit around a vertex but do not yield a semiregular tiling of the plane.
15. Know that the order of regular polygons about a vertex can affect whether or not a vertex type produces a semiregular tiling of the plane.
16. Given any triangle or quadrilateral, demonstrate an edge-to-edge, monohedral tiling of the plane using only the triangle or quadrilateral.
17. Know that only certain types of pentagons and hexagons can be used to create edge-to-edge, monohedral tilings of the plane.
18. Know that it is not possible to form monohedral tilings of the plane with a *convex*  $n$ -gon for  $n \geq 7$ .
19. Know that some *concave*  $n$ -gons with 7 or more sides can give rise to tilings of the plane.
20. Use the methods on pp. 437–439 of your text to create new shapes from rectangles and triangles that will tile the plane.
21. State Euler's Formula for Convex Polyhedra.
22. Apply Euler's Formula to find an Unknown Polyhedron.
23. Know that there are only 5 regular polyhedra: the tetrahedron, the cube, the octahedron, the dodecahedron, and the icosahedron.
24. Analyze tic-tac-toe games, Nim games, and other alternate move games using game trees, partial game trees, and oral arguments.
25. Analyze games of chance using game trees, partial game trees, and oral arguments.