Worksheet 5 Name:_____
A&S100
11 October 2002

1. Complete the addition and multiplication tables for arithmetic modulo 5. Remember that each of your answers should be $0, 1, 2, 3$, or 4.

$+$	0	1	2	3	4
0					
1					
2					
3					
4					

$\times$	0	1	2	3	4
0					
1					
2					
3					
4					

2. Complete the addition and multiplication tables for arithmetic modulo 6. Remember that each of your answers should be $0, 1, 2, 3, 4$ or 5.

$+$	0	1	2	3	4	5
0						
1						
2						
3						
4						
5						

$\times$	0	1	2	3	4	5
0						
1						
2						
3						
4						
5						

3. If there is a number b such that $a + b = 0$ then b is said to be the *additive inverse* of a. Find the additive inverse of each of the following numbers when sums are computed modulo 5 or state that none exists.

   (a) 1
   (b) 2
   (c) 3
   (d) 4

   Find the additive inverse of each of the following numbers when sums are computed modulo 6 or state that none exists.

   (a) 1
   (b) 2
   (c) 3
   (d) 4
   (e) 5

4. If there is a number b such that $a \times b = 1$ then b is said to be the *multiplicative inverse* of a. Find the multiplicative inverse of each of the following numbers when sums are computed modulo 5 or state that none exists.

   (a) 1
   (b) 2
   (c) 3
   (d) 4

   Find the multiplicative inverse of each of the following numbers when sums are computed modulo 6 or state that none exists.

   (a) 1
   (b) 2
   (c) 3
   (d) 4
   (e) 5

5. A nonzero number a is said to be a *zero divisor* if there is a number $b \neq 0$ such that $a \times b = 0$.

   (a) List all zero divisors for arithmetic modulo 5.
   (b) List all zero divisors for arithmetic modulo 6. How do the zero divisors relate to 6?
   (c) Would you guess that arithmetic modulo 7 has zero divisors? Why or why not? If you think that arithmetic modulo 7 has zero divisors, which of $0, 1, 2, 3, 4, 5$, and 6 do you suppose to be zero divisors?

(d) Would you guess that 0 modulo 9 has zero divisors? Why or why not? If you think that arithmetic modulo 9 has zero divisors, which of $0, 1, 2, 3, 4, 5, 6, 7, 8$ and 9 do you suppose to be zero divisors?

6. Find a number n that simultaneously satisfies all of the following conditions:

   (a) $2 \leq n \leq 100$,
   (b) $n \equiv 1 \pmod{2}$,
   (c) $n \equiv 1 \pmod{3}$,
   (d) $n \equiv 1 \pmod{4}$,
   (e) $n \equiv 1 \pmod{5}$, and
   (f) $n \equiv 1 \pmod{6}$

7. Can you find a number n that simultaneously satisfies all of the following conditions?

   (a) $n \geq 2$,
   (b) $n \equiv 1 \pmod{2}$,
   (c) $n \equiv 1 \pmod{3}$,
   (d) $n \equiv 1 \pmod{4}$,
   (e) $n \equiv 1 \pmod{5}$,
   (f) $n \equiv 1 \pmod{6}$,
   (g) $n \equiv 1 \pmod{7}$,
   (h) $n \equiv 1 \pmod{8}$,
   (i) $n \equiv 1 \pmod{9}$, and
   (j) $n \equiv 1 \pmod{10}$?

   Can you find such a number if we also require that $n < 3000$?