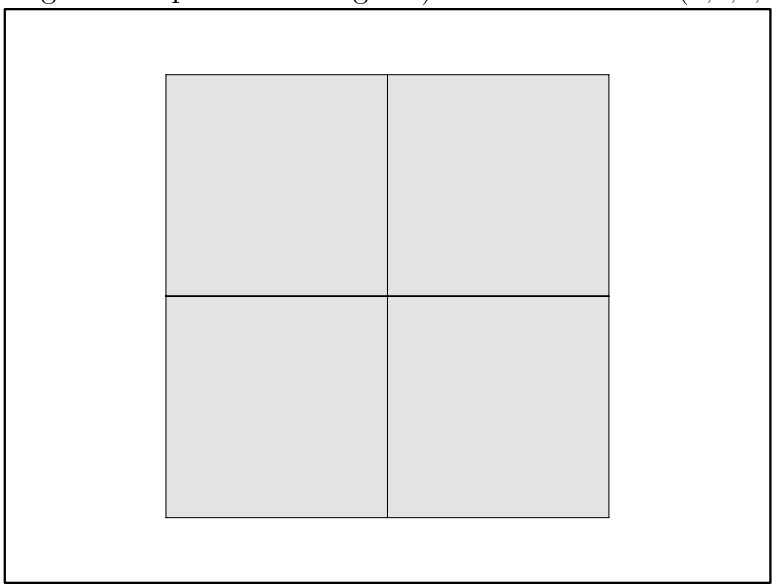
A&S 153
#1 Plane Clusters

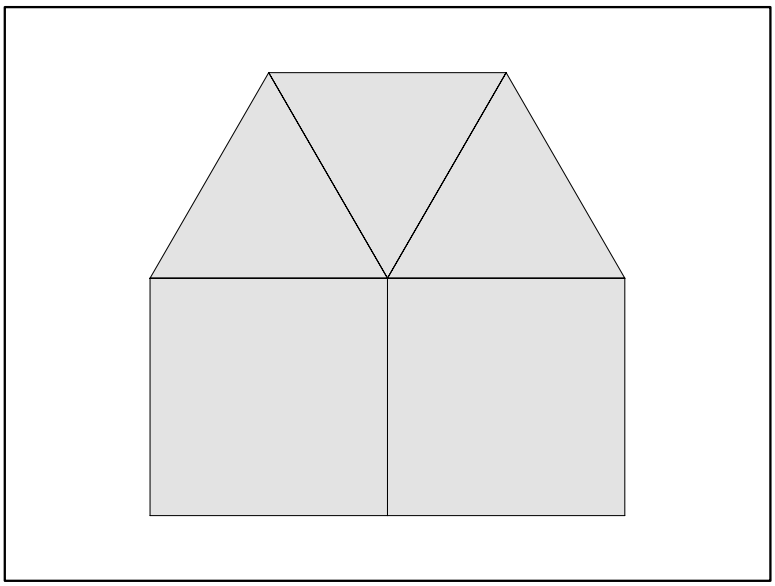
Be prepared to discuss your solutions to these problems on Wednesday, August 30. I will collect them at a later date.

Four squares can be fit together perfectly in the plane surrounding a common corner (since each interior angle of a square is 90 degrees). Let's call this a (4,4,4,4) cluster.

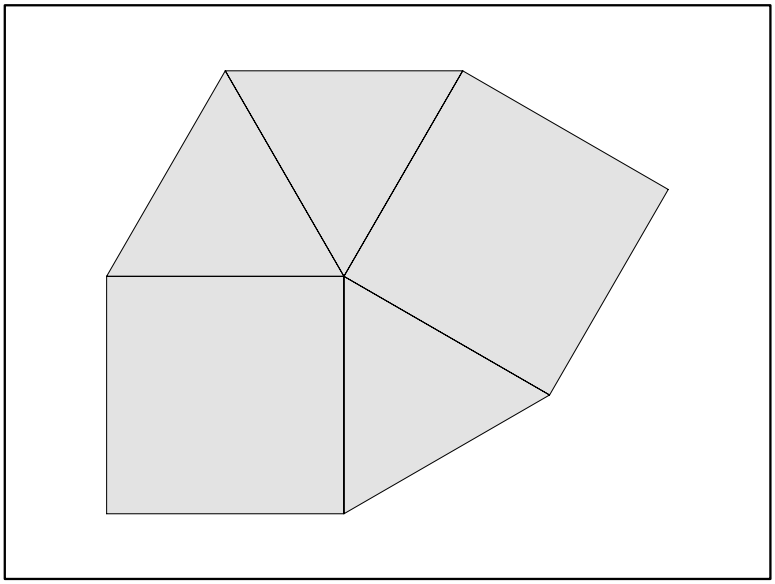


(4,4,4,4) Cluster

Similarly, two squares and three equilateral triangles can fit together perfectly surrounding a common corner. There are essentially two different ways to do this: (4,4,3,3,3) (where the squares are adjacent) and (4,3,4,3,3) (where the squares are not adjacent).



(4,4,3,3,3) Cluster



(4,3,4,3,3) Cluster

Note that we could have called this last cluster $(3, 3, 4, 3, 4)$ as well—it still refers to the same cluster. However, $(4, 4, 3, 3, 3)$ and $(4, 3, 4, 3, 3)$ are *not* the same.

1. You have just seen three clusters. Try to determine all possible clusters that can be formed by placing combinations of regular polygons in the plane surrounding a common corner. Be *systematic* in some fashion, so that you can be certain you have found all of them. You may need to determine a formula for the angle of a regular n-gon.

2. Some of the clusters can be extended to cover (tile) the plane so that at every corner point of the tiling, exactly the same cluster appears—the same sequence of polygons, in either clockwise or counterclockwise order. For example, if you tile the plane with squares, you have a $(4, 4, 4, 4)$ cluster at every single corner. Of the clusters you have found, determine which ones can be extended. Make a good drawing of each one you have found. I will include some polygons which you may use for tracing if you wish.