

## Rational and Exponential Functions, and Rational Exponents

1. Consider the Fibonacci sequence, 1, 1, 2, 3, 5, 8,... where each number  $a(n)$  is given by  $a(n)=a(n-1)+a(n-2)$ ,  $n>2$ . What kind of growth does this sequence seem to have (linear, quadratic, exponential, etc.)?
2. Consider the sequence 1, 1, 3, 5, 11,... where each number  $a(n)$  is given by  $a(n)=a(n-1)+2*a(n-1)$ ,  $n>2$ .
  - What kind of growth does this sequence seem to have (linear, quadratic, exponential, etc.)?
  - Can you find a closed formula for  $a(n)$ ?
3. How many moves does it take to solve the Towers of Hanoi puzzle with  $n$  disks? (See the National Library of Virtual Manipulatives.)
4. About 20% of the original Carbon-14 remains in a piece of wood. If the half life of Carbon-14 is 5568 years, how old is the piece of wood?
5. Suppose the velocity of a falling object after  $t$  seconds is given by the function  $v(t)=s(t) = 100(1 - e^{-0.1t})$  m/sec.
  - What is the terminal velocity?
  - When does the object reach terminal velocity?
6. The radius of the earth is 3956.6 miles. Suppose you are at the ocean, standing on a tower.
  - Express the distance to the horizon as a function of the height  $h$  of the tower.
  - How high would the tower have to be if you wanted to see 10 miles out to sea?
7. A spherical balloon is expanding. Write a formula for the surface area as a function of volume.
8. What shape rectangle encloses an area of 100 square meters and has minimum perimeter?
9. Consider a sphere of radius 1 centered at the origin, and let  $N$  be the point  $(0,0,1)$  (the "North Pole"). For any point  $P(x,y,z)$  different from  $N$ , draw the ray  $NP$ , and let  $Q(X,Y,0)$  be the point of intersection with the plane  $z=0$ .
  - Find equations to convert the coordinates of  $P$  to the coordinates of  $Q$  and vice versa.
  - Look up stereographic projection maps of the earth and discuss their advantages and disadvantages.

10. Suppose you possess a credit card with the following terms:  
<http://www.ms.uky.edu/~lee/ma111sp10/creditcard.pdf>. Suppose you buy a \$1000 TV, charge it to your card, and then each month make the minimum payment.
- How long will it take to pay off this purchase?
  - How much money will you end up paying?
11. What is the sum  $S(n)$  of the first  $n$  numbers in a geometric sequence  $a, ar, ar^2, ar^3, \dots$ ? What is its limit as  $n$  approaches infinity?