History of Mathematics #3

1. Before 11 pm, Sunday, January 29. Go to the Forum "Elements" and make at least one substantive contribution by 11 pm Sunday, January 29, and at least one substantive response to others' postings before class on Tuesday, January 31. Write about the following: When do various pieces of Euclid's *Elements* appear in current K–16 curriculum?

2. Before Tuesday, January 31.

   (a) Read Dunham Chapter 3, MTA Sketch 15, and Boyer Chapter 7 (if you haven't already).

   (b) Think about the following questions for discussion at the Centra session:

      i. Carefully study the proofs presented in Dunham and be prepared to describe them. Make some sketches in advance with Geometer's Sketchpad or Wingeom. I would welcome volunteers!

      ii. Construct a geometric model for $(a + b + c)^2$.

      iii. Construct a geometric model for $(a + b)^3$ (a good exercise for Algeblocks!).

      iv. Construct a geometric model for $1 + 3 + 5 + \cdots + (2n - 1) = n^2$.

      v. Go to the website http://www.ms.uky.edu/~lee/ma241/ma241.html and download the notes. Look at the description of a geometric model for $1^2 + 2^2 + 3^2 + \cdots + n^2$ on page 76. You may also want to download the Wingeom file "Cubedissect.wg3" from the website.

      vi. Construct a regular pentagon using Geometer's Sketchpad or Wingeom.

      vii. Given a tetrahedron, how can one inscribe a sphere within it, and circumscribe a sphere about it?

      viii. Prove the formula for the volume of a cone using calculus.

      ix. Try mapping the relevant parts of the *Elements* to a modern undergraduate text in number theory.

3. Tuesday, January 31, 7–9 pm. Attend the Centra session to discuss the readings, forum, and comments and questions on the assigned homework due on Friday.

4. Homework problems due Friday, February 3, 11 pm, uploaded on the moodle site (preferred method) or submitted to the email address mathhist@ms.uky.edu.

   (a) Even though there is an infinite number of primes, it is possible to construct long sequences of composite numbers. Consider the number $n = 1000000!$ (one million factorial). Explain why none of the numbers $n+2, n+3, n+4, \ldots, n+1000000$ is prime.

(b) Euclid's Algorithm and Continued Fractions

i. Dunham on page 69 describes Euclid's Algorithm for finding the greatest common divisor for two positive integers. He illustrates it with the numbers 1387 and 3796. Look at the sequence of quotients generated in the process, and then explain how they can be used to construct the *continued fraction* representation of $\frac{3796}{1387}$:

$$\frac{3796}{1387} = 2 + \cfrac{1}{1 + \frac{1}{2+\frac{1}{1+\frac{1}{4}}}}.$$

Start out by computing:

$$\begin{aligned} \tfrac{3796}{1387} &= 2 + \tfrac{1022}{1387} \\ &= 2 + \tfrac{1}{\frac{1387}{1022}}. \end{aligned}$$

ii. What happens if we do the above process for the two adjacent Fibonacci numbers 34 and 21?

iii. If you try this process with an irrational number, like $\sqrt{2}$, instead of a rational number like $\frac{3796}{1387}$, the following occurs:

$$\begin{aligned} \sqrt{2} &= 1 + (\sqrt{2}-1) \\ &= 1 + \tfrac{1}{\frac{1}{\sqrt{2}-1}} \\ &= 1 + \tfrac{1}{\frac{\sqrt{2}+1}{1}} \\ &= 1 + \tfrac{1}{2+(\sqrt{2}-1)} \\ &= 1 + \tfrac{1}{2+\frac{1}{\frac{1}{\sqrt{2}-1}}} \\ &\vdots \\ &= 1 + \tfrac{1}{2+\frac{1}{2+\frac{1}{2+\frac{1}{2+\ddots}}}}. \end{aligned}$$

The result is the *continued fraction* expansion of $\sqrt{2}$, which has a nice infinite repeating pattern despite the irrationality of $\sqrt{2}$.

If we did not know that this represented $\sqrt{2}$, then we could have figured it out by setting this expression equal to x, and then observing that the pattern implies that

$$x = 1 + \frac{1}{1+x}.$$

Explain why the pattern implies this equation, and then solve this equation to confirm that $x = \sqrt{2}$.

iv. Show that the following continued fraction equals the Golden Ratio:

$$1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{\ddots}}}}}.$$

(c) There are four regular solids known as the Kepler-Poinsot solids. What are they? Who discovered them and when? Why are they considered regular; i.e., why does it make sense to say that each face is the same regular polygon, and the same number of regular polygons meet at each vertex? What makes them different from the five classical regular solids, so that they were not discovered until many centuries later? If possible, include pictures—I believe they are built into the 3-d Wingeom window (try looking under "Units"), so you can cut and paste from there.