History of Mathematics #5

1. Before 11 pm, Sunday, February 12. Go to the Forum "Equations" and make at least one substantive contribution by 11 pm, Sunday, February 12, and at least one substantive response to others' postings before class on Tuesday, February 14. Write about BOTH of the following:

   (a) Regarding MTA Sketch 9: Would it be reasonable or beneficial to introduce the method of false position and double false position into the high school curriculum? When? Would it help to understand linearity?

   (b) Regarding MTA Sketch 10: High school texts often have students solve quadratic equations algebraically by writing the equation with 0 on one side, factoring, and setting each linear factor to zero to find the two roots. Completing the square comes later, often in connection with deriving the quadratic formula, usually without pictures. This approach is historically backwards. Would an historically faithful approach be preferable in high school? Should it be introduced at all? With algebra tiles? (Paraphrased from MTA, Expanded Edition, page 132.)

2. Before Tuesday, February 14.

   (a) Read Dunham Chapter 5 and MTA Sketches 8–10. Skim Boyer Chapters 9–14. We're jumping through many centuries here, and something may catch your fancy for a class presentation.

   (b) Think about the following questions for discussion at the Centra session:

      i. Carefully study the mathematics in Dunham and in MTA. In particular, be sure you can explain the method of false position and the method of double false position, as well as the geometric approach described to solving the quadratic equation in MTA Sketch 10.

      ii. Is there a three-dimensional analog of Heron's formula for the volume of a tetrahedron in terms of its six side lengths? In terms of its four face areas?

      iii. Look up the Heron-like formula for the area of a cyclic quadrilateral. To whom is this attributed? How is it proved?

      iv. Look up the definitions of conic sections in terms of their foci and directrices. Derive equations for them from these definitions.

      v. What properties of reflection do the conic sections have? How are these proved?

vi. From MTA, Expanded Edition, page 126: "In order to solve equations algebraically, we need to use several abstract ideas: a symbol for the unknown, zero, negative numbers, the notion of compensating operations on the two sides of an equation." How do the methods of false position sidestep the need for such concepts? "Does this make it easier or harder to learn and remember how to solve linear equation problems?"

vii. From MTA, Expanded Edition, page 126: "Robert Recorde's *The Whetstone of Witte* is written as a dialogue between a master and his student. After the master explains the basic idea behind setting up and solving equations, the student says:

> "It seemeth that this rule is all one with the rule of false position, and therefore might so be called, seeing it taketh a false number to worke withal."

"The master replies, however, that this method 'doeth not take a false number, but a true number for his position, as it shall be declared anon,' and adds that it 'teacheth a man at the first word to name a true number before he knoweth resolutely what he hath named.'

"What is the point of the distinction the master makes? What does it say about the algebraic approach to solving problems?"

viii. From MTA, Expanded Edition, page 126: "*Daboll's Schoolmaster's Assistant* was the most widely used arithmetic book in the United States in the first half of the 19th century. It was published by many different companies for more than 40 years, starting in 1799." Why was this book so popular?

ix. From MTA, Expanded Edition, page 131: "Al-Khwārizmī classified [quadratic] equations into six types. Three of them involve only two terms (e.g., squares are equal to things), and three involve more than two (e.g., a square and things equal to numbers). Explain why he needed to do this, and list (in modern form) all six types of equations."

3. Tuesday, February 14, 7–9 pm. Attend the Centra session to discuss the readings, forum, and comments and questions on the assigned homework due on Friday.

4. Homework problems due Friday, February 17, 11 pm, uploaded on the moodle site (preferred method) or submitted to the email address mathhist@ms.uky.edu.

   (a) Explain why it is only necessary to go to the square root of the highest number in the grid when doing the sieve method to find primes. Do not use outside sources for this problem, though you may talk with each other.

(b) Find an alternative algebraic proof for Heron's formula in the following way: Let ΔABC be a triangle with side lengths $a = BC$, $b = AC$, and $c = AB$. Assume c is the larger of these three numbers, and position the triangle with $\overline{AB}$ as the base. Choose D on $\overline{AB}$ so that $\overline{CD}$ is the altitude. Let $h = CD$, $x = AD$, and $c - x = BD$. Now use the Pythagorean theorem, algebra, and perseverance to show that $\frac{1}{2}ch$ equals Heron's formula. Do not use outside sources for this problem, though you may talk with each other.

(c) From MTA, Expanded Edition, page 125: "Use double false position to solve the following problem, which is based on a problem in chapter 12 of Leonardo of Pisa's *Liber Abbaci*.

> "A certain man proceeding to Lucca on business to make a profit doubled his money, and he spent there 12 denari. He then left and went through Florence; he there doubled his money, and spent 12 denari. Then he returned to Pisa, doubled his money, and spent 12 denari. At the end of the three trips and after the expenses there will be 9 denari. It is sought how much he had at the beginning."

Do not use outside sources for this problem, though you may talk with each other.

(d) From MTA, Expanded Edition, page 125: "In the explanation of false position given above [i.e., in Sketch 9], there are two cases: either the two errors are of the same kind, or they are not. Then the text says 'This is just a way of avoiding negative numbers.' Explain." Do not use outside sources for this problem, though you may talk with each other.

(e) From MTA, Expanded Edition, page 131: "The geometric argument described in this sketch [Sketch 10] only works for 'a square and things equal to numbers,' that is, for equations of the form $x^2 + bx = c$ (with b and c positive). Develop an analogous geometric argument to solve 'a square is equal to things and numbers,' that is, an equation of the form $x^2 = bx + c$." Illustrate your method with the equation $x^2 = 4x + 12$. Do not use outside sources for this problem, though you may talk with each other.

Extra Credit: "Can you do the same for 'a square and numbers is equal to things'? (Be careful, this is significantly more complicated.)" Do not use outside sources for this problem, though you may talk with each other.