Some Notes on Homework #9

1. The cycloid problem. Here is a way to derive the equation of the cycloid that is equivalent to what all of you did, but perhaps a bit shorter (including no need for multiple cases):

   First, assume the circle is centered at $(0,0)$ and calculate the coordinates of the point (x,y) resulting from the clockwise (negative) rotation of the point $(0,-a)$ (hence initially at an angle of $-\pi/2$) through the angle z:

   $$(x,y) = (a\cos(-\pi/2 - z), a\sin(-\pi/2 - z) = (-a\sin z, -a\cos z)$$

   Now translate the circle up by a units and to the right by az units to put it in its final position:

   $$(x,y) = (-a\sin z, -a\cos z) + (az, a) = (az - a\sin z, a - a\cos z).$$

2. The tautochrone.

   One minor point in physics: Because rolling objects have kinetic energy associated with both the rolling (rotational kinetic energy) as well as the velocity along the curve, I am not sure if the tautochrone (or the brachistochrone) property works for rolling objects (but I admit that I have not done the calculations). Ideally we must consider frictionless sliding objects.

3. The bricks.

   If you poke around on the web, you can find a photo of bricks with large overhang. For example, I googled "harmonic brick photo" and immediately found

   www.antiquark.com/2005/04/harmonic-series-and-bricks.html.

   Amazing what you can find on the web!

4. The non-transitive dice.

   Here is how I remember the numbering for the dice. Begin with the standard 3×3 magic square:

8	1	6
3	5	7
4	9	2

You can use the numbers in the first row for die A, just using every number twice: 1, 1, 6, 6, 8, 8. Similarly for dice B and C. But to avoid repetition of numbers on the dice, I doubled the numbers, and subtracted 1 from one of the repetitions: 1, 2, 11, 12, 15, 16 for die A, etc.