MA111 – Homework #4 Short Solutions

Chapter 6

8. (a) A,B,C,F,E,D,A; A,D,F,E,B,C,A; A,B,E,D,F,C,A; A,D,E,F,C,B,A; A,C,B,E,F,D,A; A,C,F,D,E,B,A.
   (b) D,A,B,C,F,E,D; D,F,E,B,C,A,D; D,F,C,A,B,E,D; D,E,F,C,B,A,D; D,A,C,B,E,F,D; D,E,B,A,C,F,D.

12. (a) One answer is: D,A,G,F,E,C,B.
    (b) One answer is: B,A,G,F,E,C,D.
    (c) Any Hamilton path must pass through at least two of B, D, and G, and therefore must include at least two of the edges AB, AD, and AG. But if the path begins at A, then it must include only one edge touching A, so this would be impossible. A symmetrical argument shows that starting the path at C also is impossible.
    (d) Any Hamilton circuit must pass through B, D, and G, and therefore must include the edges AB, AD, and AG—more than two edges touching A. So this is impossible.

16. (a) One answer is: B,A,E,C,D with weight=19.
    (b) One answer is: B,A,C,E,D with weight=25.
    (c) You must list and check all possible Hamilton paths starting at B and ending at D: B,A,C,E,D; B,A,E,C,D; The best one is B,A,E,C,D with weight=19.

18. (a) You can divide 20! by 20 to get 19!, so the answer is 121,645,100,408,832,000.
    (b) $\frac{20!}{19!} = \frac{20\cdot19\cdot18\cdot17\cdots3\cdot2\cdot1}{19\cdot18\cdot17\cdots3\cdot2\cdot1} = 20.$
    (c) $\frac{201!}{199!} = \frac{201\cdot200\cdot199\cdot198\cdot197\cdots3\cdot2\cdot1}{199\cdot198\cdot197\cdots3\cdot2\cdot1} = 201\cdot200 = 40,200.$

22. You can use www.wolframalpha.com for these, or your calculator to get an approximation.
    (a) $25! = 15,511,210,043,330,985,984,000,000 \approx 1.55\times10^{25}$.
    (b) $27! = 10,888,869,450,418,352,160,768,000,000 \approx 1.09\times10^{28}$.
    (c) 25!, which equals the number in part (a).

24. (a) $\approx 491,857$ years (using 365 days in a year) or $\approx 491,521$ years (using 365.25 days in a year). Calculation in the latter case: 25! circuits $\times \frac{1\text{ sec}}{10^{12}\text{ circuits}} \times \frac{1\text{ min}}{60\text{ sec}} \times \frac{1\text{ hr}}{60\text{ min}} \times \frac{1\text{ day}}{24\text{ hr}} \times \frac{1\text{ yr}}{365.25\text{ days}}$.
    (b) $\approx 12,788,288$ years (using 365 days in a year) or $\approx 12,779,500$ years (using 365.25 days in a year).

26. (a) $\frac{200\times199}{2} = 19,900$ edges.
    (b) $\frac{201\times200}{2} = 20,100$ edges.
    (c) $\frac{501\times500}{2} - \frac{500\times499}{2} = 500.$

28. (a) If $(N-1)! = 720$, what is N? By checking low values you can see that $6! = 720$. So $N-1=6$. Therefore $N=7$.
    (b) If $\frac{N(N-1)}{2} = 66$, what is N? Multiplying by 2 we have $N(N-1) = 132$. N should be close to the square root of 132, which is between 11 and 12. Checking that $12(11) = 132$ we see that $N = 12$ is a solution.
    (c) If $\frac{N(N-1)}{2} = 80,200$, what is N? Multiplying by 2 we have $N(N-1) = 160,400$. N should be close to the square root of $160,400$, which is between 400 and 401. Checking that $401(400) = 160,400$ we see that $N = 401$ is a solution.

34. (a) E,C,I,G,M,T,E with travel time 19.4 years.
    (b) T,M,I,C,G,E,T. Rewrite starting from E: E,T,M,I,C,G,E. Travel time is 18.9 years.
    (c) You need to check all the valid possibilities: E,G,I,M,C,T,E; E,G,I,M,T,C,E; E,G,M,I,C,T,E; E,G,M,I,T,C,E; E,I,G,M,C,T,E; E,I,G,M,T,C,E; E,I,M,G,C,T,E; E,I,M,G,T,C,E; E,M,G,I,C,T,E; E,M,G,I,T,C,E; E,M,I,G,C,T,E; E,M,I,G,T,C,E. The best one is E,G,I,M,T,C,E with travel time 18.3 years.

38. You need to run the Nearest Neighbor algorithm starting at each possible vertex, find the cheapest one, and then rewrite that one starting at D. Solution: D,A,F,E,B,C,D with cost $200.